Three-flow model of nonuniformly fluidized beds in the system gas—particles

[a]O. MIKULA and [b]I. HAVALDA

[a]*Department of Chemical Engineering, Refinery and Petrochemical Company, Slovnaft n.e., 823 00 Bratislava*

[b]*Department of Chemical Engineering, Slovak Technical University, 880 37 Bratislava*

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In this paper three regions found experimentally for the expansion of nonuniformly fluidized beds are characterized. A theoretical model of the nonuniformly fluidized bed in the system gas—particles is described and an equation is derived for the relative expansion of bed. The values of relative expansions calculated from this equation are compared with the experiment in the system air—particles and with the values calculated according to the equation put forward by *Pyle* and *Harrison* [1]. Our equation is in better agreement with experiment than the equation of *Pyle* and *Harrison*.

In previous paper [2] we judged the two-phase model of nonuniformly fluidized beds proposed by *Pyle* and *Harrison* [1]. In this connection we have presented some experimental data, which we used to elaborate and verify the so-called three-flow model of nonuniformly fluidized beds in the system gas (at low pressures)—particles.

Discussion

In a nonuniformly fluidized bed appearing in systems gas (at low pressure)—particles the aggregates of particles arise by the effect of disturbance forces [3]. In these aggregates the particles touch each other. During its existence, the aggregate behaves as a separate particle and the parameters characteristic of particle can be attributed to it. Only a small part of the fluid passes through the interior of the aggregate.

According to the experimental data, there are three regions which may be distinguished at a relative expansion of nonuniformly fluidized beds. This statement is confirmed by Fig. 2 in our preceding paper [2].

In the first region, limited by the interval $Re_i \leq Re < Re_{c,max}$, the relative maximum height L_{max}/L_0 and the relative minimum height L_{min}/L_0 increase slowly with the value of Re. The bed height fluctuates, but it is relatively distinctly confined and the number of particles shot over that demarkation is practically negligible. The visual observation showed that moving bubbles were observed only when the values of Re came near to the value $Re_{c,max}$.

In the second region, limited by the interval $Re_{c,max} \leq Re < Re_{c,min}$, the relative maximum height L_{max}/L_0 increases with the value of Re much more rapidly than it does at $Re < Re_{c,max}$, but the relative minimum height L_{min}/L_0 changes as in the first region. The bed height fluctuates considerably and is indistinct while the number of particles shot over the level increases with Re. An intensive formation of bubbles was observed visually.

In the third region, limited by the interval $Re \geqslant Re_{c,min}$, the values L_{max}/L_0 and L_{min}/L_0 increase with Re almost with the same rate as L_{max}/L_0 in the second region. The formation of bubbles is very intensive, the size of bubbles is comparable with the diameter of equipment but no pistons arise. In the second region it was possible to determine visually (at least approximately) the boundary between dense suspension and particles shot over it; however, it was not possible in the third region. Owing to a great number of bubbles the number of particles in a volume unit did not differ much from the number of particles in a volume unit of the space filled with the particles shot out. Nevertheless, there is a qualitative difference between the bed and the space filled with the particles shot out. Whereas there are bubbles of gas, aggregates of particles, and separate particles in the bed, no bubbles appear in the space filled with the particles shot out.

The idea that in a dense suspension the aggregates of particles, the separate particles, and the gas bubbles occur is the basis of the three-flow model. The separate particles and the aggregates will be denoted as effective particles.

The aggregates are of different size. Since the magnitude of disturbance forces under otherwise identical conditions is a quasi-stationary stochastic process [3] it may be assumed that

a) the mass fraction of the particles forming aggregates (and thus also the mass fraction of separate particles in bed) is a quasi-stationary stochastic process;

b) the number of aggregates in bed is a quasi-stationary stochastic process.

If we choose a convenient measure for the size of aggregates, d_z, we can in every moment distinguish the size range of effective particles in bed which is also a quasi-stationary stochastic process.

The maximum range of the sizes of effective particles which has been found may then be separated into m equally wide intervals. The mean value $d'_{e,i}$ as a class sign, *i.e.* the characteristic size of effective particles may then be attributed to the i-th interval. It is evident that for a certain $d'_{e,i}$ a mass fraction $\bar{x}_i$ of effective particles exists in the i-th interval of sizes and this fraction is a quasi-stationary stochastic process.

Let a certain aggregate of particle be of the volume V_a

$$V_a = n_a V_p + V_{ga} = \frac{n_a V_p}{1 - \varepsilon_a}, \tag{1}$$

where n_a is the number of the equisized particles of equal density ϱ_g in a certain aggregate of particles, V_p is the volume of one particle, V_{ga} is the volume of the fluid of density ϱ_g in an aggregate of particles, and ε_a is the porosity inside aggregate defined by the ratio V_{gp}/V_a. The term equivalent diameter of the aggregate of particles d_z stands for the diameter of such a sphere which is of equal volume V_a as a certain aggregate of particles. Since in a certain moment it is not possible to determine the ratio of the number of separate particles to the number of aggregates, we shall assume that the bed consists only of effective particles with effective diameter d_{ze} defined by

$$d_{ze} = \frac{1}{\sum_{i=1}^{m} x_i/d'_{e,i}} \tag{2}$$

Evidently d_{ze} changes with time as a quasi-stationary stochastic process.

The value of porosity, ε_a, inside an aggregate changes from $\varepsilon_a = 0$ when the aggregate consists only of one particle up to $\varepsilon_a \doteq 0.42$ which corresponds to the porosity at incipient fluidization of spherical particles approximating an orthorhombic arrangement [4].

The definition of effective diameter according to equation (2) leads to an idea of a bed with particles of different density, which has not been explained mathematically so far. The calculation density of the aggregate with a certain effective diameter appeared to be a problem. For further considerations we, therefore, introduced the concept of the so-called modified diameter of effective particles d_z^*. This is the diameter of a spherical particle of density ϱ_p identical with the density of the particles forming a bed at which the bed consisting of these imaginary particles would show for $w > w_i$ equal pulsation and average value of the height of dense suspension as the real bed for which d_z^* is defined under otherwise identical conditions.

If d_{ze} changes with time as a stationary stochastic process, then also d_z^* changes in this manner because both are conditioned by the change in the height of dense suspension which is a quasi-stationary stochastic process. With respect to the limited magnitude of disturbance forces under otherwise constant conditions [3] an upper limit of the quantity d_z^* exists, which will be denoted as $(d_z^*)_{max}$.

On the basis of the elimination method according to *Beňa* [5] we assume that in systems fluidized by gas $(d_z^*)_{max}$ is a function of the characteristic length dimension of particles d_e (which are assumed to be of equal size), the height of compact bed of particles L_0, compressibility of gas β, the effective weight of unit volume of the particle $g(\varrho_p - \varrho_g)$, the density ϱ_g and dynamic viscosity of gas μ, the incipient fluidizing velocity w_i, the superficial gas velocity w, the diameter of equipment D, and the construction of grid expressed by means of the parameter K. We shall assume that this parameter is dimensionless and its value will not depend on w. Dimensional analysis of the set of quantities

$$\{(d_z^*)_{max}, d_e, L_0, \beta, g(\varrho_p - \varrho_g), \varrho_g, \mu, w_i, w, D, K\} \tag{3}$$

led to the following equation among dimensionless products:

$$\frac{(d_z^*)_{max}}{d_e} = f_1\left(Re_i, Re, Ar, \pi_\beta, \frac{D}{d_e}, \frac{L_0}{D}, K\right), \tag{4}$$

where

$$\pi_\beta = \frac{\beta\, \mu_g^2}{\varrho_g\, D} \tag{5}$$

The equation (4) may be simplified on the basis of the following considerations and assumptions.

1. It is known that Re_i is an unambiguous function of Ar.

2. For $D/d_e > 10$, the effect of the walls of equipment on hydrodynamic regime is in uniformly fluidized beds negligible. For the nonuniformly fluidized beds we shall assume that the effect of D/d_e (in our experiments $D/d_e \geqslant 248.9$) is also negligible and the effect of walls represents the simplex L_0/D.

3. For measurements in the same system, *i.e.* for $D =$ const, $p =$ const (where p is the pressure in system), $t =$ const and thus $\varrho_g =$ const, $\mu =$ const it holds $\pi_\beta =$ const even for the criterion defined by equation (5).

4. We proved [3] that in the range of the data worked up by us it is valid $L_{min}/L_0 \neq f(L_0)$ and $L_{max}/L_0 \neq f(L_0)$, *i.e.* for $D = \text{const}$ it may be assumed that the simplex L_0/D in equation (4) will not influence the value $(d_z^\bullet)_{max}$.

5. The value K is considered to be constant in measurements in the same system. Therefore the equation (4) may be transformed to the form

$$\frac{(d_z^\bullet)_{max}}{d_e} = f_2(Re_i, Re). \tag{6}$$

Furthermore, we assume that in the interval $Re_i \leq Re \leq Re_{c,min}$ (*i.e.* in the first and second expansion region) the effective particles are at the moment when the relative height of bed attains the value L_{min}/L_0 fluidized equally as at incipient fluidization while the modified diameter $d_z^\bullet$ reaches the instantaneous maximum value $(d_z^\bullet)_{max}$. The observed moderate increase in the height of L_{min}/L_0 with respect to L_i/L_0 (Fig. 2 in paper [2]) in the interval $Re_i < Re \leq Re_{c,min}$ must be explained by the irregularity of the shape of aggregates.

Assuming that the value of Archimedes number $(Ar_z^\bullet)_{max}$

$$(Ar_z^\bullet)_{max} = (d_z^\bullet)^3_{max} \frac{g(\varrho_p - \varrho_g)\,\varrho_g}{2} \tag{7}$$

satisfies the inequality $(Ar_z^\bullet)_{max} \leq 10^5$ (what in the range of the data of our experiments may be expected because of $Ar \in \langle 197; 8955 \rangle$), the equation derived by *Beňa et al.* [6—8] which is valid for particles of various shape

$$Re_i = 0.00138\, Ar^{0.890} \quad (\text{if } 200 < Ar \leq 10^5) \tag{8}$$

may be applied to the calculation of $R^{\cdot}_i$ in the case of a bed containing effective particles with a modified maximum diameter $(d_z^\bullet)_{max}$. Thus it may be written

$$(Re_{iz}^\bullet)_{max} = 0.00138\, (Ar_z^\bullet)^{0.890}_{max}, \tag{9}$$

where

$$(Re_{iz}^\bullet)_{max} = \frac{(w_{iz}^\bullet)_{max}\, \varrho_g}{\mu}\, (d_z^\bullet)_{max}. \tag{10}$$

By inserting from equations (7) and (10) in (9) and rearranging we obtain

$$(d_z^\bullet)_{max} = \left[(w_{iz}^\bullet)_{max} \frac{B}{0.00138\, A}\right]^{0.599} \tag{11}$$

where

$$A = \left[\frac{g(\varrho_p - \varrho_g)\,\varrho_g}{2}\right]^{0.890} \tag{12}$$

$$B = \frac{\varrho_g}{\mu}. \tag{13}$$

For individual series of measurements [3] with the samples of particles B_1, B_2, B_3, and B_4 (Table 1, paper [2]) the values $(d_z^\bullet)_{max}$ were calculated according to equation (11) using the values $(w_{iz}^\bullet)_{max}$ taken from the interval $\langle w_i; w_{c,min} \rangle$. Further elaboration

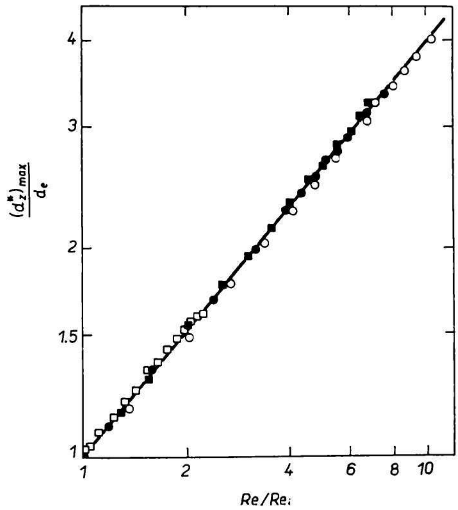

Fig. 1. Graphical representation of the function $\frac{(d_z^*)_{max}}{d_e} = f\left(\frac{Re}{Re_i}\right)$.
○ sample of particles B_1; ● sample of particles B_2; ■ sample of particles B_3; □ sample of particles B_4.
——— graphical representation of the empirical equation (*14*).

of the calculated values $(d_z^*)_{max}$ has shown (Fig. 1) that the empirical equation of the form

$$\frac{(d_z^*)_{max}}{d_e} = 1.000 \left(\frac{Re}{Re_i}\right)^{0.590} \tag{14}$$

complies with the functional relation (*6*). If we substitute for Re_i in (*14*) from (*8*) we obtain

$$\frac{(d_z^*)_{max}}{d_e} = 48.69 \left(\frac{Re}{Ar^{0.890}}\right)^{0.590} \tag{15}$$

The basic ideas of the three-flow model may be formulated as follows.

1. A nonuniformly fluidized bed in the system gas—particles consists of two regions, *i.e.* a dense suspension and gas bubbles.

In a dense suspension the separate particles and the aggregates of particles occur. For the probability mean quantities characterizing the hydrodynamic behaviour of nonuniformly fluidized beds (systems gas at low pressures—particles) the characteristic length dimension $(d_z^*)_s$ (so-called modified mean diameter of effective particles) is determining. It shows the character of effective diameter and is defined by equation

$$(d_z^*)_s = \frac{(d_z^*)_{max} + \lambda d_e}{1 + \lambda}, \tag{16}$$

where λ is a parameter which has not been determined yet. All bubbles are of equal volume V_b (cm³), defined [9] by equation

$$V_b = 0.806 \left(\frac{Q_b^2}{g}\right)^{3/5} \tag{17}$$

(where $[Q_b]$ = cm³/s, $[g]$ = cm/s²) and an equal rising velocity u_b [cm/s] defined [10, 11] by equation

$$u_b = 0.792\, g^{0.5}\, V_b^{1/6} \tag{18}$$

(where $[V_b] = cm^3$; $[g] = cm/s^2$).

2. For a certain system one value $(d_z^*)_{max}$ or $(d_z^*)_s$ is attached to each value $Re \geq Re_i$. With increasing Re, the value $(d_z^*)_{max}$ increases according to equations (*14*) or (*15*) whereas the value $(d_z^*)_s$ increases according to equation (*16*).

3. The cross section of bed S may be divided in the cross section S_d through which the gas enters the dense suspension and the cross section S_B through which the gas flows in the form of bubbles. It holds

$$S = S_d + S_B. \tag{19}$$

4. The total flow-rate of gas Q branches in three flows at the inlet into bed. In the first flow the amount $(Q_{iz}^*)_a$ enters the bed through the cross section S_d and fluidizes the particles of diameter $(d_z^*)_s$ equally as at incipient fluidization. The porosity of the bed containing these particles (diameter $(d_z^*)_s$) is, in general, different from that of the whole bed at incipient fluidization. In the second flow the amount $(Q')_a$ enters the bed through the cross section S_d and goes through the aggregates of particles. In the third flow the amount $(Q_b)_a$ enters the bed through the cross section S_B and is the source of bubble formation. The material balance of gas may then be expressed by equation

$$Q = (Q_{iz}^*)_a + (Q')_a + (Q_b)_a. \tag{20}$$

The disadvantage of this model consists in the fact that it is not possible to perform a direct experimental verification of the applicability of equations (*16*) and (*20*).

On the basis of the above model the mean height L_a may be expressed as follows.

The flow-rates in the equation (*20*) express according to their definition the following relations:

$$Q = w\, S, \tag{21}$$

$$(Q_{iz}^*)_a = (w_{iz}^*)_s\, S_d = (w_{iz}^*)_s\, (\alpha\, S), \tag{22}$$

$$(Q_b)_a = u_b\, S_B = u_b[(1 - \alpha)\, S], \tag{23}$$

where $(w_{iz}^*)_s$ is the incipient fluidizing velocity of the bed corresponding to the particles of diameter $(d_z^*)_s$ and

$$\alpha = \frac{S_d}{S} \leq 1. \tag{24}$$

The equality in the relationship (*24*) is valid only for the incipient fluidization.

The flow-rate of gas $(Q')_a$ which goes through the aggregates of particles may be assumed to equal the difference of the flow-rate which belongs to the particles of diameter $(d_z^*)_s$ at incipient fluidization for the cross section of column S and the flow-rate $(Q_{iz}^*)_a$. Thus it holds

$$(Q')_a = (w_{iz}^*)_s\, S - (w_{iz}^*)_s\, (\alpha\, S) = (w_{iz}^*)_s\, [(1 - \alpha)\, S]. \tag{25}$$

The quantity α may be expressed independently on the basis of the following consideration. Let the fluidized bed be of the height L_a at a certain value $w \geqslant w_i$; then the volume of dense suspension V_d will be

$$V_d = S_d\, L_a. \tag{26}$$

At incipient fluidization the volume of dense suspension V_{di} is identical with the total volume of bed V_i.

$$V_{di} = V_i = L_i S = 1.724 L_0 S, \tag{27}$$

provided that the particles are spherical and $\varepsilon_i = 0.420$. If we neglect the change in the porosity at incipient fluidization among the aggregates of particles, the following equation will be valid because of the change in their shape

$$V_d = V_{di} \tag{28}$$

from which after inserting from equations (*26*) and (*27*) and respecting equation (*24*) we obtain

$$\alpha = 1.724 \frac{L_0}{L_a}. \tag{29}$$

The velocity of gas w_B in dilute suspension referred to the cross section S_B is identical with the rising velocity of one bubble u_b. If we substitute in equation (*18*) for V_b from equation (*17*) and $(Q_b)_a \equiv Q_b$ express by means of equations (*20*), (*21*), (*22*), and (*25*) we get after rearrangement

$$u_b = 11.45\, D^{2/5}[w - (w^{*}_{iz})_s]^{1/5}, \tag{30}$$

(where $[w] = [w^{*}_{iz}]_s = [u_b] = $ cm/s, $[D] = $ cm).

After substituting the acceleration due to gravity g and rearranging in an equation without dimensional constant equations (*21*), (*22*), (*23*), (*25*), (*29*), and (*30*) give for L_a

$$\frac{L_a}{L_0} = \frac{1.724(g\,D)^{2/5}}{(g\,D)^{2/5} - 1.373[w - (w^{*}_{iz})_s]^{4/5}} \tag{31}$$

The value $(w^{*}_{iz})_s$ may be calculated from equation (*8*) if we put $d = (d^{*}_z)_s$ and $w_i = (w^{*}_{iz})_s$. Thus we obtain

$$(w^{*}_{iz})_s = (d^{*}_z)_s^{1.670} \left(\frac{0.00138\,A}{B}\right), \tag{32}$$

where A and B are defined by equations (*12*) and (*13*). We failed to put forward a complete concept of the quality λ in equation (*16*). Further we shall assume a symmetrical distribution of particles according to size; hence $\lambda = 1$.

By substituting for $(d^{*}_z)_s$ from (*16*) in (*32*) and regarding (*15*) we obtain after rearranging for $\lambda = 1$ the following equation without any dimensional constant

$$(w^{*}_{iz})_s = 0.00138\, Ar^{0.890} \left(\frac{\mu}{d_e\,\varrho_g}\right) \left\{0.5 \left[1 + 48.69 \left(\frac{Re}{Ar^{0.890}}\right)^{0.590}\right]\right\}^{1.67} \tag{33}$$

For $Re = Re_i$, *i.e.* $(d^{*}_z)_s = d_e$ the expression in braces of the equation (*33*) equals one and it holds $(w^{*}_{iz})_s = w_i$.

Figs. 2—5 present some data on relative expansion $L_a/L_0 = f(Re)$ (full lines) which have been calculated on the assumption that $\lambda = 1$ according to equations (*31*) and (*33*) for a series of measurements with the samples of particles B_1, B_2, B_3, and B_4 fluidized by air [2]. It is obvious that the agreement between the values of relative expansion

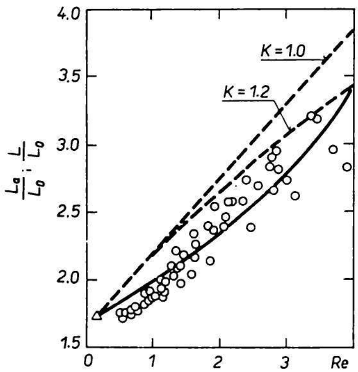

Fig. Confrontation of the relative expansions L_a/L_0 calculated from equation (*31*) ——— with experimental values and with relative expansions L/L_0 according to *Pyle* and *Harrison* [1] — — — for a sample of particles B_1 ($Ar = 197$) fluidized by air.
△ incipient fluidization; experimental values.

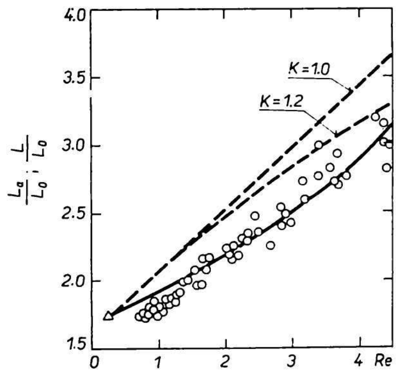

Fig. 3. Confrontation of the relative expansions L_a/L_0 calculated from equation (*31*) ——— with experimental values and with relative expansions L/L_0 according to *Pyle* and *Harrison* [1] — — for a sample of particles B_2 ($Ar = 350$) fluidized by air.
△ incipient fluidization; experimental values.

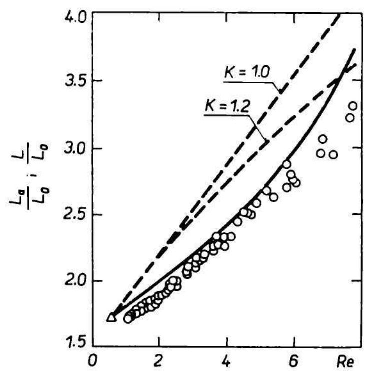

Fig. 4. Confrontation of the relative expansions L_a/L_0 calculated from equation (*31*) ——— with experimental values and with relative expansion L/L_0 according to *Pyle* and *Harrison* [1] — — — for a sample of particles B_3 ($Ar = 908$) fluidized by air.
△ incipient fluidization; ○ experimental values.

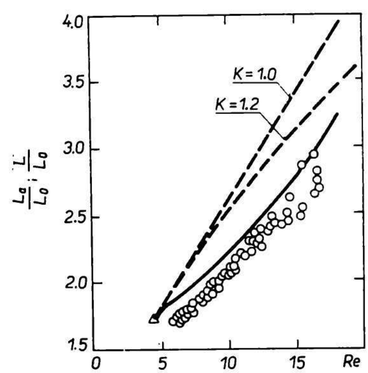

Fig. 5. Confrontation of the relative expansions L_a/L_0 calculated from equation (*31*) ——— with experimental values and with relative expansion L/L_0 according to *Pyle* and *Harrison* [1] — — — for a sample of particles B_4 ($Ar = 8955$) fluidized by air.
△ incipient fluidization; experimental values.

L_a/L_0 calculated according to equations *(31)* and *(33)* and the experimental values is relatively good. The maximum percentual deviation of the values L_a/L_0 according to equation *(31)* with respect to the smoothed experimental data is 14%.

It is evident from the course of theoretical curves and the set of experimental points that the quantity λ is at least a function of the gas velocity and the size of particles while the effect of particle size is more marked (it determines the systematic deviation). For sufficient understanding of λ further experimental data are necessary. It may be supposed that λ is a function of the conveniently defined Froude number which characterizes in a sense the degree of inhomogeneity of fluidized bed.

For comparison, Figs. 2—5 also show (dashed lines) the equations of relative expansion $L/L_0 = f(Re)$ where the value L was calculated from the following equation derived by *Pyle* and *Harrison* [1] according to two-phase model of the nonuniformly fluidized bed

$$\frac{L_a}{L} = 1 - \frac{w - w_i}{K(w - w_i) - u_{bi}} \qquad (34)$$

The agreement between the experimental values of relative expansion and the values calculated according to equation *(31)* is in the whole range of Re and for all samples of particles investigated much better than it is in the case of the values calculated according to equation *(34)* derived by *Pyle* and *Harrison*.

In conclusion it must be pointed out that equations *(31)* and *(33)* were verified only for $Ar \in \langle 197; 8955 \rangle$ if air at atmospheric pressure was used as a fluidizing gas.

Furthermore it would be convenient to examine the effect of different compressibility β, of quantity L_0/D, and of different kinds of grids.

Symbols

Ar	$= g\, d^3 (\varrho_p - \varrho_g)\, \varrho_g\, \mu^{-2}$ — Archimedes number
$(Ar_z^*)_{max}$	Archimedes number defined by the relationship *(7)*
d_e	characteristic length dimension of particle
d_{ze}	effective diameter of effective particles defined by the relationship *(2)*
d_z^*	modified diameter of effective particles
$(d_z^*)_{max}$	maximum modified diameter of effective particles
$(d_z^*)_s$	characteristic length dimension defined by equation *(16)*
D	inner diameter of column
g	acceleration due to gravity
K	parameter characterizing the grid or dimensionless coefficient in equation *(34)*
L	fluidized bed height
L_{max}, L_{min}	maximum and minimum height of the nonuniformly fluidized bed at the time of observation under constant conditions
L_0	$= 4m/\pi D^2 \varrho_p$ — height of compact bed of particles
L_i	bed height at incipient fluidization
L_a	$= \dfrac{L_{max} + L_{min}}{2}$ — average height of fluidized bed
m	mass of the sample of particles
n_a	number of particles in an aggregate
Re	$= w\, d_e\, \varrho_g\, \mu^{-1}$ — Reynolds number
$Re_{c,max}$	Reynolds number characterizing the break of the curve $\log (L_{max}/L_0) = f(\log Re)$

$Re_{c,min}$	Reynolds number characterizing the break of the curve $\log (L_{min}/L_0) = f(\log Re)$
Re_i	$= w_i d_e \varrho_g \mu^{-1}$ — Reynolds number at incipient fluidization
$(Re^{*}_{iz})_{max}$	Reynolds number defined by relationship *(10)*
S	cross section of column
S_B	part of the cross section of column S through which gas enters the dilute suspension
S_d	part of the cross section of column S through which gas enters the dense suspension
Q	total flow-rate of gas through the bed
Q_b	flow-rate of gas in the bubble phase through the bed
$(Q')_a$	average flow-rate of gas passing through aggregates of particles
$(Q_b)_a$	average flow-rate of gas effected by bubbles
$(Q^{*}_{iz})_a$	average flow-rate of gas attached to the particles of $(d^{*}_z)_s$ diameter at incipient fluidization
	rising velocity of a separate bubble in the vicinity of the incipient fluidization
u_b	rising velocity of bubbles through bed
V_b	bubble volume
V_p	particle volume
V_{ga}	volume of gas in an aggregate of particles
V_d	volume of dense suspension at $w > w_i$
V_i	volume of bed at incipient fluidization
V_a	volume of an aggregate of particles
	superficial gas velocity
w_i	incipient fluidizing velocity
$(w^{*}_{iz})_{max}$	incipient fluidizing velocity attached to the effective particles of $(d^{*}_z)_{max}$ diameter
$(w^{*}_{iz})_s$	incipient fluidizing velocity attached to the effective particles of $(d^{*}_z)_s$ diameter
α	parameter defined by relationship *(24)*
β	compressibility of gas
ε	porosity of fluidized bed
	porosity of bed at incipient fluidization
ε_a	porosity inside an aggregate of particles
λ	undefined parameter in equation *(16)*
	dynamic viscosity of gas
π_β	dimensionless complex defined by relationship *(5)*
	density of gas
	density of particles

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