

## Hydrodynamic properties of rotary-disc extractor. II. Longitudinal mixing in continuous phase

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Received 10 May 1976

Accepted for publication 23 September 1976

A mathematical model of the mechanism of longitudinal mixing in continuous phase in an RD extractor is presented. It enables us to calculate the coefficient of longitudinal mixing  $\epsilon_c$  on the basis of diffusion model for the design of these extractors. The constants in the relationship derived were determined experimentally by measurements in a laboratory extraction column containing the system water—acetone—toluene. The values of  $Pe_c$  ascertained by means of concentration profiles were compared with the values calculated according to the recommended correlation relationships.

Разработана математическая модель механизма продольного перемешивания в сплошной фазе РД-экстрактора, которая позволяет на основании диффузионной модели рассчитывать коэффициент продольного перемешивания  $\epsilon_c$  для нужд проектировки этих экстракторов. Константы в полученном окончательном соотношении были получены экспериментальным путем на лабораторной экстракционной колонне для системы вода—ацетон—толуол. Значения  $Pe_c$ , полученные из концентрационных профилей, были сравнены с рассчитанными по рекомендованным корреляционным соотношениям.

At present the behaviour of countercurrent extractors is described by means of two mathematical and physical models, *i.e.* the diffusion model and the stage model which are used for the design of these devices. The diffusion model is based on the idea of differential phase contact and interfacial as well as intrafacial substance transfer in each phase. This intrafacial substance transfer is due to the hydrodynamic character of two-phase flow and may be expressed quantitatively in the form of the first Fick's law

$$J_i = -\epsilon_i \frac{dc_i}{dh} \quad (1)$$

where the coefficient of turbulent diffusion  $\epsilon_i$  is denoted as the coefficient of longitudinal mixing. The fundamental equations of the diffusion model (Fig. 1) are

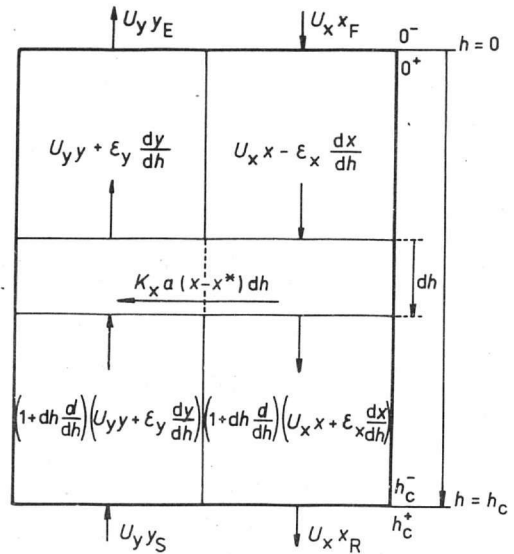


Fig. 1. Diagram of diffusion model of countercurrent extractor.

$$\epsilon_x \frac{d^2 x}{dh^2} - U_x \frac{dx}{dh} - K_x a (x - x^*) = 0 \quad (2)$$

$$\epsilon_y \frac{d^2 y}{dh^2} + U_y \frac{dy}{dh} + K_x a (x - x^*) = 0 \quad (3)$$

with the boundary conditions according to *Danckwerts* [1] respecting the concentration discontinuities at the entrance of both phases into apparatus

$$h = 0^+; \quad U_x x_F = U_x x - \epsilon_x \frac{dx}{dh}; \quad \frac{dy}{dh} = 0;$$

$$h = h_c^-; \quad U_y y_S = U_y y + \epsilon_y \frac{dy}{dh}; \quad \frac{dx}{dh} = 0 \quad (4)$$

On certain simplifying presumptions (constant values of the quantities involved and linear equilibrium dependence) it is possible to find an analytical solution of eqns (2—4) which was published by several authors [2—5] and which gives the concentration profile of the penetrating substance in apparatus. The procedure put forward by *Rod* [6] which consists in introducing the modified concentrations  $X$  and  $Y$

$$X = x - \frac{\epsilon_x}{U_x} \frac{dx}{dh} \quad (5)$$

$$Y = y + \frac{\epsilon_y}{U_y} \frac{dy}{dh} \quad (6)$$

is useful in practice.

Thus eqn (2) may be simplified into the form

$$\frac{dX}{dh} = -\frac{K_x a}{U_x} (x - x^*) \quad (7)$$

and boundary conditions (4) assume the following form

$$\begin{aligned} h = 0^+; \quad x = x_0; \quad X = x_F; \quad y = Y = y_E; \\ h = h_c^-; \quad x = X = x_R; \quad y = Y = y_S \end{aligned} \quad (8)$$

From the balance of a column section of limited length the following equation may be obtained

$$U_x X - U_y Y = U_x x_F - U_y y_E \quad (9)$$

On replacing differentials by differences, we are able by using eqns (5–7), the known form of equilibrium dependence, and eqns (8) and (9) to determine the so-called balance line of apparatus  $y = f(x)$  and its length numerically. The merit of this procedure consists in that it makes possible an explicit calculation of  $h_c$ , may be used for arbitrary form of equilibrium dependence, and does not require constant values of influential quantities. Moreover, eqns (5–9) enable us from experimental concentration profile in apparatus to determine the values of  $\varepsilon_x$ ,  $\varepsilon_y$ , and  $K_x a$  in the form of dimensionless criteria of  $Pe_x$ ,  $Pe_y$ , and  $No_x$ . By integrating eqns (5–7) along the whole apparatus and respecting eqn (8) we obtain

$$Pe_x = \frac{U_x h_c}{\varepsilon_x} = \frac{x_0 - x_R}{\int_0^1 (X - x) dz} \quad (10)$$

$$Pe_y = \frac{U_y h_c}{\varepsilon_y} = \frac{y_E - y_H}{\int_0^1 (y - Y) dz} \quad (11)$$

$$No_x = \frac{K_x a h_c}{U_x} = \frac{x_F - x_R}{\int_0^1 (x - x^*) dz} \quad (12)$$

in which the integrals may be calculated graphically. The values of  $X$  are to be obtained from integrated eqn (7)

$$x_F - X_z = No_x \int_0^z (x - x^*) dz \quad (13)$$

while eqn (9) gives the corresponding values of  $Y$ . Still, it is necessary to know the equilibrium dependence  $y^* = f(x)$ .

The knowledge of the coefficients of longitudinal mixing is a precondition for the use of models to design and for this reason, attention has been paid to the determination of these quantities. The indirect methods of stationary and nonstationary analysis which are widely used in this field are based on the observation of

the response to a signal in the form of stationary or nonstationary feed of the tracing substance into apparatus and the properties of this response earmark the method of determination of the value  $Pe$  expressing the character of flow [1, 7—11]. The starting relationships are the equations of diffusion model adjusted to the conditions under which the tracer has been applied. In this way some correlation equations for the calculation of  $Pe$  from the quantities determining the flow character have been derived. Because of intricate experimental technique attention was prevalingly concentrated on one-phase flow or continuous phase of two-phase flow. The relationships under discussion involve the *Strand* equation [10]

$$\frac{n}{Pe_c} (1 - Z) = 0.5 + 0.09(1 - Z) \frac{D_r N}{U_c} \left[ \frac{D_r}{D_c} \right]^2 \left[ \frac{D_s^2 - D_r^2}{D_c^2} \right] \quad (14)$$

the *Miyauchi* equation [12]

$$\frac{n}{Pe_c} = 0.5 + 0.017 \frac{D_r N(1 - Z)}{U_c n} N_p^{1/3} \left[ \frac{D_c}{h_m} \right]^{1/2} \quad (15)$$

and the *Mišek* equation [13]

$$\frac{n}{Pe_c} = 0.5 + k_0 \frac{D_r N(1 - Z)}{U_c N} \left( \frac{N_p D_r^2 D_s^6}{D_c^8 h_m} \right)^{1/3} \quad (16)$$

where  $k_0 = 15.77 \times 10^{-2} \text{ cm}^{1/3}$  and

$$N_p = \frac{P_d}{N^3 D_r^3 \rho_c} = 0.069 \left( \frac{\rho_c D_r^2 N}{\mu_c} \right)^{-0.155} \quad (17)$$

However, the validity of these relationships has not been verified by means of the values of  $Pe$  obtained under equal conditions immediately from concentration profiles which are important from the view-point of designing and any relationship for the calculation of  $Pe$  based on the analysis of flow character have not been published, too.

### Theoretical

The model of the mechanism of longitudinal mixing in both phases is based on the idea of intrafacial turbulent diffusion according to eqn (1). Supposing the resulting flow character is given by an additive effect, for instance the effect of two factors and the contribution of each factor may be quantitatively characterized by the coefficient  $\varepsilon_1$  or  $\varepsilon_2$ , then eqn (1) may be expressed in the form

$$J_i = -(\varepsilon_1 + \varepsilon_2) \frac{dc_i}{dh} \quad (18)$$

and after multiplying with  $1/h_c U$

$$\frac{1}{Pe} = \frac{1}{Pe_1} + \frac{1}{Pe_2} \quad (19)$$

From the physical view-point respecting the character of the factors under discussion it is suited to write eqn (19) in the following form

$$\frac{1}{Pe} = K_0 + \frac{1}{Pe_1} + \frac{1}{Pe_2} \quad (20)$$

On the basis of the analysis of the flow of continuous phase in extractor it may be assumed that the longitudinal mixing is given by summing up the mechanical stirring of rotor discs and the mixing which takes place in the drifts arising behind the individual drops of dispersed phase during their countercurrent motion. Hence eqn (20) may be written in the form

$$\frac{1}{Pe_c} = K_0 + \left[ \frac{1}{Pe} \right]_{\text{stir}} + \left[ \frac{1}{Pe} \right]_{\text{drift}} \quad (21)$$

For further determination of the term  $[1/Pe]_{\text{stir}}$  we may start from the relationship between diffusion model and stage model which for sufficient number of stages assumes the form

$$\frac{1}{Pe} \doteq \frac{r}{n} \quad (22)$$

where the so-called coefficient of reverse mixing  $r$  in the sense of stage model is defined as the ratio of the velocity of reverse flow  $U_{c,z}$  to the velocity of main flow  $U_c$  of continuous phase. For  $U_{c,z}$  it holds

$$U_{c,z} = U_z(1 - Z) \quad (23)$$

The quantity  $U_z$  is the velocity of total reverse flow depending on the number of revolutions  $N$  and diameter  $D_r$  of rotor according to [14]

$$U_z = K_1 N D_r \quad (24)$$

and  $Z$  is the hold-up of dispersed phase defined as the ratio of the volume of dispersed phase in the working part of column to the whole volume of this part. It may be calculated from the equation proposed by Míšek [15]

$$\frac{U_d}{Z} + \frac{U_c}{1 - Z} = \kappa u_0 \Gamma(Z) \quad (25)$$

where  $\Gamma(Z)$  is the known function of hold-up, hydrodynamic conditions, and physical properties of system and  $\kappa$  is the relative free cross-section of column.

By combining eqns (22—24) we obtain

$$\left[ \frac{1}{Pe} \right]_{\text{stir}} = K_1 \frac{D_r N(1-Z)}{U_c n} \quad (26)$$

The resulting effect of mixing in drifts behind drops must depend on the number of individual drops in a given cross-section of column and their velocity. With increasing number of drops (increasing hold-up) more drifts are formed, but the diameter as well as fall velocity of drops usually decreases which results in a decrease of the intensity of mixing. It means that two opposite effects are simultaneously in operation. The mean velocity of drop fall may be characterized by means of the dimensionless mean residence time in column  $\bar{\theta}$  defined by the equation

$$\bar{\theta} = \frac{U_d \bar{t}}{h_c} \quad (27)$$

where  $\bar{t}$  is the first initial moment of the residence time distribution

$$\bar{t} = \int_0^{\infty} t k(t) dt \quad (28)$$

and  $k(t)$  is the frequency function for this distribution.  $\bar{t}$  represents a fictitious mean residence time which does not respect the effect of drop interaction. Because of these facts it may be assumed that  $[1/Pe]_{\text{drift}}$  depends on  $\bar{\theta}$  and for the lack of convenient data this relationship was chosen in the form

$$\left[ \frac{1}{Pe} \right]_{\text{drift}} = K_2 \left[ \frac{\bar{t} U_d}{h_c} \right]^{K_3} \quad (29)$$

The value of the exponent  $K_3$  indicates which of both above-mentioned effects prevails. By substituting eqns (26) and (29) into eqn (21) we obtain a general expression for  $Pe_c$

$$\frac{1}{Pe_c} = K_0 + K_1 \frac{D_r N(1-Z)}{U_c n} + K_2 \left[ \frac{\bar{t} U_d}{h_c} \right]^{K_3} \quad (30)$$

The constant  $K_0$  determines the value of  $Pe$  if both the subsequent terms are equal to zero (one-phase flow without mechanical stirring). The form of the function  $k(t)$  necessary for the calculation of  $\bar{t}$  according to eqn (28) may be found out from the known drop size distribution  $f(d)$  and the relationship between the velocity of fall  $u$  and the drop diameter  $g(d)$ . For the residence time of a drop it holds

$$t_i = \frac{h_c}{u_i} = \frac{h_c}{g(d_i)} \quad (31)$$

and hence according to [16]

$$k(t) = f \left[ g^{-1} \left( \frac{h_c}{t} \right) \right] \left| \left[ g^{-1} \left( \frac{h_c}{t} \right) \right] \right| \quad (32)$$

where  $g^{-1}$  is an inversion function with respect to the function  $g$ .

The constants of eqn (30) were found experimentally using the data obtained by evaluating the concentration profiles in the extractor.

### Experimental

A laboratory rotary-disc extraction column and the system water (continuous phase)—acetone—toluene (dispersed phase), where toluene simultaneously proved as depleted phase [17], were used for experiments. The concentration profile of acetone in steady state in the column was determined as follows. The wall of column was drilled through on seven equidistant spots and the sampling devices were put into the holes. These devices enabled us

Table 1

Comparison of the experimental values of  $Z$ ,  $No_x$ ,  $\bar{t}$ ,  $Pe_c$  with the calculated values by means of correlation relations

Experiment	$Z$	$Pe_y = Pe_c$	$No_x$	$\bar{t}$ s	$Pe_c$ (14)	$Pe_c$ (16)	$Pe_c$ (15)
1	0.095	8.4	1.38	6.16	38.7	23.8	24.9
2	0.114	12.3	1.70	6.90	36.8	23.0	24.2
3	0.129	15.0	2.37	7.50	35.2	22.4	23.5
4	0.168	19.1	2.93	8.26	33.2	22.2	23.3
5	0.232	32.0	5.25	9.54	30.7	22.4	23.5
6	0.098	9.8	1.46	6.06	36.6	22.0	23.2
7	0.104	15.2	1.79	6.16	35.1	21.1	22.2
8	0.138	18.7	2.37	7.20	33.1	20.8	21.9
9	0.171	20.3	3.39	9.65	31.4	20.5	21.6
10	0.219	18.9	4.51	10.96	29.2	20.4	21.5
11	0.230	22.9	5.91	10.39	27.6	19.5	20.5
12	0.116	8.5	1.56	6.50	34.0	20.3	21.4
13	0.125	11.6	1.76	7.07	32.4	19.5	20.6
14	0.161	14.4	2.15	8.07	30.5	19.3	20.3
15	0.172	16.6	2.55	9.68	29.2	18.6	19.6
16	0.216	21.2	3.65	12.57	27.2	18.5	19.5
17	0.250	31.8	4.33	12.85	25.3	18.0	19.0
18	0.128	10.1	1.58	6.59	30.2	17.6	18.5
19	0.143	12.8	2.01	7.71	28.6	16.9	17.8
20	0.163	17.0	2.22	8.43	27.1	16.4	17.4
21	0.186	57.0	2.85	8.96	25.7	16.1	17.0
22	0.225	25.2	3.21	11.96	24.0	15.8	16.7
23	0.260	23.0	3.89	—	22.3	15.5	16.4
24	0.117	8.3	1.56	6.04	28.9	16.3	17.2
25	0.154	11.4	1.83	7.58	27.1	16.0	16.9
26	0.171	13.4	2.23	7.94	25.8	15.5	16.4
27	0.188	18.1	2.44	9.01	24.4	15.1	15.9

to take off the required amount of either phase separately while the hydrodynamic regime in column was influenced only slightly. The devices consisted of injection needles terminated by cocks on the outside and the so-called sampling testpieces on the inside. For the withdrawing of continuous phase these testpieces were of brass in the shape of drilled cylinders while for the withdrawing of dispersed phase they were of polyethylene in the shape of small pipes. For withdrawing, the different wettability of brass and polyethylene with respect to water and toluene was utilized. As for the sampling itself, it was necessary to take care that the samples contained only pure phases and no further substance transfer occurred in the sample and no acetone evaporated from the sample. For obtaining representative samples of dispersed phase two sampling devices were always placed at each withdrawing spot, the first at the wall of column and the second one in the proximity of disc. Moreover, samples were taken from the outgoing water and toluene phase during each experiment (working conditions of all experiments are given in Table 1 of the preceding paper [17]). The inlet concentration of organic phase varied about the value of  $107 \text{ kg m}^{-3}$  and pure water was used for extraction. The mean temperature of both phases in the column was  $19^\circ\text{C}$ . For this temperature the following experimental equilibrium relationship was found

$$y = 13.7 + 1.11 x - 1.08 \times 10^{-3} x^2 \quad (33)$$

Four samples at the average were taken at each spot. In each experiment a set of 80–90 samples was taken and the concentration of acetone in samples was determined refractometrically. The arithmetic means found for individual places of the column were used for the determination of the concentration profile expressed as a dependence of acetone concentration on the position given by the distance from the inlet of depleted phase.

In each experiment the value of hold-up was also determined and the values obtained are given in Table 1. It appeared that the determination of concentration profile by means of samples was possible only under those hydrodynamic conditions when the hold-up was large enough to obtain the samples of required properties, as regards the dispersed phase, but smaller than the critical value of hold-up at congestion.

## Results and discussion

A typical concentration profile found in the extractor is shown in Fig. 2 (experiment 15 of Table 1). The values of  $Pe_c$  and  $No_x$  for each experiment were calculated from the concentration profiles by means of eqns (10), (12), and (13) and are quoted in Table 1. They were used for the determination of the constants  $K_0$ – $K_3$  in eqn (30). It was proved experimentally [17] that the drop size distribution under the experimental conditions used could be described by the Schwarz equation [18]

$$f(d) = \frac{B}{d^5} \exp\left(-\frac{\bar{a}}{d}\right) \quad (34)$$

where



$$B = \frac{\bar{a}^4 \exp\left(\frac{\bar{a}}{d_m}\right)}{6\left(1 + \frac{\bar{a}}{d_m} + \frac{\bar{a}^2}{2d_m^2} + \frac{\bar{a}^3}{6d_m^3}\right)} = \frac{\bar{a}^4 \exp\left(\frac{\bar{a}}{d_m}\right)}{G} \quad (35)$$

The dependence  $g(d)$  of the fall velocity of drop  $u$  on its diameter  $d$  corresponds to the transitory region of flow, as demonstrated in [19], and may be expressed by the Allen equation [20]

$$u = 0.153 \frac{d^{1.14} \Delta \rho^{0.71} g^{0.71}}{\rho_c^{0.29} \mu_c^{0.43}} \doteq k_2 d = g(d) \quad (36)$$

The expression behind the sign of approximate equality was chosen in the above form because it made possible to obtain further relationships in a relatively simple form. This sign does not pass in these relationships and the differences caused by replacing the exponent 1.14 by the unit value (the exponent over  $d$ ) are in the sense of model involved in the constants  $K_2$  and  $K_3$ . By substituting eqns (34–36) into eqns (32) and (38) we obtain

$$\bar{t} = \frac{k_2^4 \bar{a}^4 \exp\left(\frac{\bar{a}}{d_m}\right)}{h_c^4 G} \int_{t_{\min}}^{\infty} t^4 \exp\left(-\bar{a} k_2 \frac{t}{h_c}\right) dt \quad (37)$$

The integration limits express the interval of residence times of drops with the diameters from 0 to  $d_m$ . After rearranging we obtain

$$\bar{t} = \frac{24h_c}{k_2 \bar{a} G} \left(1 + \frac{\bar{a}}{d_m} + \frac{\bar{a}^2}{2d_m^2} + \frac{\bar{a}^3}{6d_m^3} + \frac{\bar{a}^4}{24d_m^4}\right) \quad (38)$$

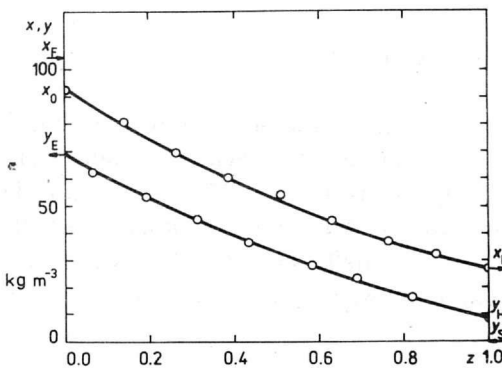


Fig. 2. Concentration profile determined in experiment 15.

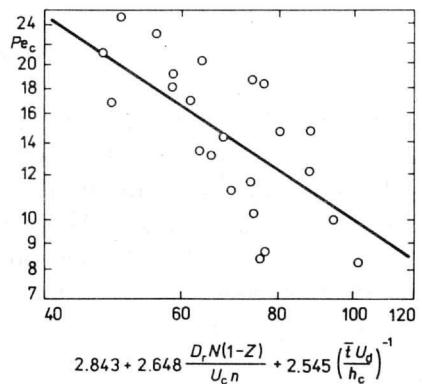


Fig. 3. Confrontation of the measured values of  $Pe_c$  with eqn (39).

The values of  $\bar{t}$  are given for individual experiments in Table 1. On calculating the constants by the method of least squares eqn (30) assumes the following form

$$\frac{1}{Pe_c} = 10^{-3} \left( 2.84 + 2.65 \frac{D_r N(1-Z)}{U_c n} + 2.55 \left[ \frac{\bar{t} U_d}{h_c} \right]^{-1} \right) \quad (39)$$

$$\delta = 0.2563, \delta_m = 0.8769,$$

where  $\delta$  and  $\delta_m$  stand for the mean and maximum relative error of individual calculated values  $Pe_c$  with respect to the values measured [17]. The confrontation of the measured and calculated values of  $Pe_c$  is presented in Fig. 3.

On the basis of dimensional analysis and general form of the criterial function adapted to the experimental conditions described in [17], the dependence of  $Pe_c$  on significant variable quantities was derived

$$Pe_c = 3.1 \times 10^{-7} \alpha^{-0.443} A^{1.73} Re^{1.34} \quad (40)$$

$$\delta = 0.1060, \delta_m = 0.3052.$$

The relatively high values of  $\delta$  and  $\delta_m$  in eqn (39) are due to some values of  $Pe$  which are quite different from the others. By omitting them these errors drop to the values  $\delta = 0.2066$  and  $\delta_m = 0.5427$ . The exponent  $-1$  in the mentioned relationship indicates that the intensity of mixing in the drifts behind drops and not the number of these drifts has predominant influence on the longitudinal mixing in continuous phase.

When compared with recommended correlation relations, eqn (39) contains one surplus term which is substantiated only for two-phase flow. The application of the correlation relations derived for one-phase flow is not admissible in these cases because there is only approximate analogy between one-phase and two-phase flow, as regards hydrodynamic regime.

Expression (40) gives a better correlation of the measured values of  $Pe_c$  than eqn (39), but it must be taken into consideration that the latter results from an attempt to master this problem in a more general theoretical manner and in addition, it contains several quantities which are not directly measurable and their estimation is laden with incident error. As the experiments were carried out only with single geometrical arrangement of the working part of column and with one system of two solvents, we cannot fully appreciate the validity of the idea leading to eqn (39) on the basis of the results obtained. According to the model used the influence of these two factors on the values of the constants  $K_0$ — $K_3$  should not be significant and could appear only in drop size distribution.

The difficulty in the use of eqn (39) consists in the calculation of  $\bar{t}$  by the above method which necessitates the knowledge of the parameters of the drop size distribution. From this point of view eqn (40) is more serviceable, but its validity is equally limited.

The experimental values of  $Pe_c$  were compared with the values calculated according to correlation relations (14—16) which are also given in Table 1. This comparison shows that the calculated values of  $\epsilon_c$  are smaller than the experimental ones, which may be explained by that the relationships used are concerned only with the description of the effect of mechanical stirring and disregard the effect of the drops of dispersed phase.

It also ensues from Table 1 that the value of  $\epsilon_c$  decreases with increasing number of revolutions, which may be attributed to the fact that the hold-up and drop number increase with the value of  $N$  but the drop diameter decreases. Therefore the mean residence time of drops in apparatus rises, *i.e.* the value of  $[1/Pe]_{\text{drift}}$  falls. Since the quantity  $[1/Pe]_{\text{stir}}$  slightly increases, it may be assumed that the influence of drops on the character of flow prevails over the influence of mechanical stirring in the region of the experiments carried out.

### Symbols

$a$	specific surface
$\bar{a}$	parameter of the drop size distribution (34)
$A = \frac{D_r N(1-Z)}{U_c}$	quantity in eqn (40)
$B$	quantity defined by eqn (35)
$c$	concentration
$d$	diameter of drop
$d_m$	maximum diameter of drop
$D_r$	rotor diameter
$D_s$	stator diameter
$D_c$	column diameter
$f(d)$	frequency function of the drop size distribution according to number
$g(d)$	dependence of the fall velocity of drop on drop diameter
$G$	quantity defined by eqn (35)
$h$	distance from the origin of column
$h_c$	length of column
$h_m$	height of stirring stage
$J_i$	flow density of the $i$ -th component
$K$	total coefficient of substance transfer
$k, K$	constants
$k(t)$	distribution of the residence times of drops
$n$	number of stirring stages
$N$	number of rotor revolutions per time unit
$No_i = K_i \frac{a h_c}{U_i}$	number of transfer units of apparatus
$N_p$	quantity defined by eqn (17) (input criterion)
$P_d$	power input per one stirring stage of column

$Pe_i$	Péclet criterion defined by eqns (10) and (11)
$Re = \frac{\rho_c h_c U_c}{\mu_c(1-Z)}$	Reynolds criterion in eqn (40)
$t$	time
$\bar{t}$	mean value defined by eqn (28)
$U$	velocity of phase
$u$	fall velocity of drop
$u_0$	mean fall velocity of drops
$x$	concentration of depleted phase
$X$	concentration defined by eqn (5)
$y$	concentration of enriched phase
$Y$	concentration defined by eqn (6)
$z = h/h_c$	dimensionless distance
$Z$	hold-up of dispersed phase
$\alpha = \frac{U_d(1-Z)}{U_c Z}$	quantity in eqn (40)
$\delta$	relative error
$\varepsilon$	coefficient of longitudinal mixing
$\kappa = \frac{D_s^2}{D_c^2}$	relative free cross-section of column
$\bar{\theta}$	mean dimensionless residence time
$\rho$	phase density
$\Delta\rho$	difference between the densities of continuous and dispersed phase
$\mu$	viscosity (dynamical)

### Indices

c	continuous phase
d	dispersed phase
E	exit of enriched phase
F	entrance of depleted phase from outside
H	entrance of enriched phase from inside
$i$	$i$ -th drop, component, phase
O	entrance of depleted phase from inside
R	exit of depleted phase
S	entrance of enriched phase from outside
x	depleted phase
y	enriched phase
z	reverse flow, value in the distance $z$ from the entrance of depleted phase

### Exponents

\* equilibrium concentration

## References

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Translated by R. Domanský