

# Hydrodynamic properties of rotary-disc extractor. III. Longitudinal mixing in dispersed phase

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The mathematical model of longitudinal mixing in continuous phase in an RD-extractor based on the diffusion model has been modified with respect to the conditions characterizing the flow and properties of dispersed phase. The constants in the resulting relation derived which make possible to calculate the coefficient of longitudinal mixing in this phase  $\epsilon_d$  for the design of extractors have been determined experimentally by measurements in a laboratory extraction column with the system water—acetone—toluene. The values of  $Pe_d$  found from the concentration profiles in this apparatus are compared with the values calculated according to some recommended correlations. The possibility of indirectly determining the coefficient of longitudinal mixing in continuous phase  $Pe_c$  by the use of nonstationary analysis in two-phase flow is discussed.

Математическая модель продольного перемешивания в сплошной фазе РД-экстрактора, основанная на диффузионной модели, была модифицирована с учетом условий характеризующих поток и свойства дисперсной фазы. Константы, содержащиеся в конечной формуле для расчета коэффициента продольного перемешивания в этой фазе  $\epsilon_d$  для нужд проектировки данного типа экстракторов, были определены экспериментально на лабораторной экстракционной колонне для системы вода—ацетон—толуол. Значения  $Pe_d$  полученные из концентрационных профилей колонны, были сравнены с рассчитанными значениями по рекомендованным соотношениям корреляции. Обсуждалась возможность определения коэффициента продольного перемешивания в сплошной фазе  $Pe_c$  косвенно при помощи нестационарного анализа в двухфазном потоке.

The investigation of the properties of countercurrent extractors is founded on two model concepts, *i.e.* the diffusion model and the stage model. The diffusion model is based on a differential contact of phases while the stage model is based on the concept of a cascade of ideal mixing stages with reverse mixing between them. The flow character in extractor represents a transient state between piston flow and ideal mixer and the degree of its approximation to these limiting cases is to be

expressed either by the coefficient of longitudinal mixing  $\varepsilon$  for the diffusion model or by the coefficient of reverse mixing  $r$  for the stage model.

The fundamental equations of the diffusion model as well as the methods of solution are presented elsewhere [1]. In order to use some of the solutions for designing we must know the values of the coefficients of longitudinal mixing in both phases. That may be achieved, as described in [12], e.g. by a direct evaluation of the relations between the concentration of the transferred substance in both phases and the position in the apparatus (concentration profiles) by using the following equations

$$Pe_x = \frac{U_x h_c}{\varepsilon_x} = \frac{x_0 - x_R}{\int_0^1 (X - x) dz} \quad (1)$$

$$Pe_y = \frac{U_y h_c}{\varepsilon_y} = \frac{y_E - y_H}{\int_0^1 (y - Y) dz} \quad (2)$$

$$x_F - X_z = No_x \int_0^z (x - x^*) dz \quad (3)$$

$$No_x = \frac{K_x a h_c}{U_x} = \frac{x_F - x_R}{\int_0^1 (x - x^*) dz} \quad (4)$$

$$U_x X - U_y Y = U_x x_F - U_y y_E \quad (5)$$

and the equilibrium relationship  $y^* = f(x)$ . The meaning of the quantities in this system of equations is evident from Fig. 1 in which the concentration profile of transferred substance in both phases is represented by the dashed line for piston flow and the full line for flow with longitudinal mixing. Because of experimental

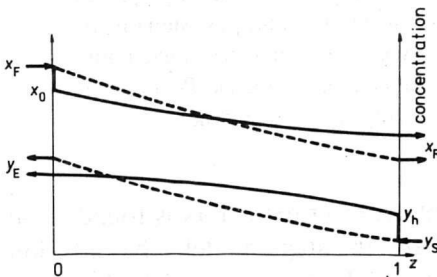


Fig. 1. Two types of the concentration profiles in rotary-disc extractor: real profile (full line), at piston flow (dashed line).

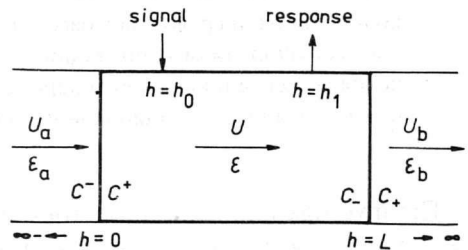


Fig. 2. Diagram of diffusion model with three mixing parts.

intricacy of this procedure, especially as regards the dispersed phase, the indirect methods of stationary and nonstationary analysis are more frequently used. These methods are based on a time-dependent or independent feed of the so-called tracing substance into the flow in apparatus and on observing "the response" at another point in the form of the concentration of this substance dependent on time and position in the apparatus. From the properties of the measured response, the value of  $Pe$  may be determined for real hydrodynamic flow conditions because these methods are founded on the diffusion model equations adapted for the conditions suited to the use of a tracing substance. For nonstationary method, it is the equation

$$\frac{\partial c}{\partial t} = \varepsilon \frac{\partial^2 c}{\partial h^2} - U \frac{\partial c}{\partial h} \quad (6)$$

the solution of which depends, in general, on boundary conditions of the model (Fig. 2) provided a concentration impulse simulating the Dirac  $\delta$  function is used as a feed of tracing substance

$$\begin{array}{ll} t \geq 0 & c = 0 \\ t = 0 & c = \infty \end{array} \quad \int_{-\infty}^{+\infty} c(t) dt = 1 \quad (7)$$

If only the middle part ( $h = 0$ ,  $h = L$ ,  $\varepsilon_a = \varepsilon_b = 0$ ,  $U_a = U_b = U$ , two-sided bounded model) is taken into consideration, the boundary conditions according to Danckwerts stated in [1] may be used for the solution. For the half-infinite model ( $\varepsilon_a = 0$ ,  $\varepsilon_b = \varepsilon$ ) and feed of tracing substance into the entry of apparatus *Strand et al.* derived the following equation [2], valid for the time dependence of  $c$  at the point  $h = L$

$$c(\vartheta) = \frac{1}{\pi} \sqrt{\frac{Pe}{\vartheta}} \exp\left[-\frac{Pe}{4}\left(\frac{1}{\sqrt{\vartheta}} - \sqrt{\vartheta}\right)\right] - \frac{Pe}{2} \exp(Pe) \operatorname{erfc}\left[\frac{\sqrt{Pe}}{2}\left(\frac{1}{\sqrt{\vartheta}} + \sqrt{\vartheta}\right)\right] \quad (8)$$

The dispersion of this equation  $\sigma^2$  (defined as the second central moment by eqn (21)) is governed by the equation

$$\sigma^2(\vartheta) = \frac{1}{Pe^2} (2Pe + 3) \quad (9)$$

where the dimensionless time is

$$\vartheta = \frac{t U}{L} \quad (10)$$

For the dispersion of the function  $c(\vartheta)$  in the bounded model (Fig. 3), *Van der Laan* [3] derived the expression

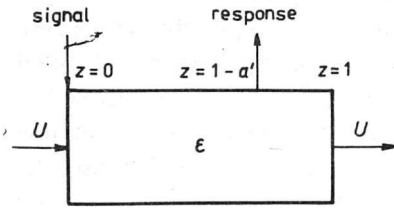


Fig. 3. Diagram of two-sided bounded diffusion model.

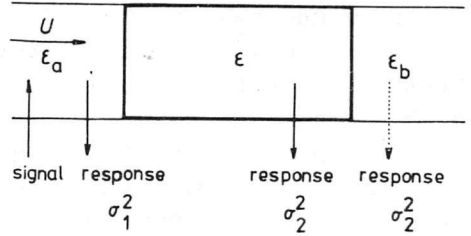


Fig. 4. Measurement of the residence time dispersion of tracing substance according to Bischoff and Levenspiel.

$$\sigma^2(\vartheta) = \frac{1}{Pe^2} [2Pe + 3 + 2e^{-Pe} - e^{-a'Pe}(4a'Pe + 4 + e^{-a'Pe})] \quad (11)$$

Bischoff and Levenspiel [4] developed a method for the determination of  $Pe$  by measuring the dependence  $c(\vartheta)$  at two different points of apparatus and simultaneously using an arbitrary concentration impulse as input signal while one or both these points could be situated outside the testing part of apparatus (Fig.4). For a two-sided infinite apparatus, these authors derived the relation

$$\sigma_2^2(\vartheta) - \sigma_1^2(\vartheta) = \frac{2}{Pe} \quad (12)$$

As a matter of fact, the experimental relationships  $c(\vartheta)$  found at the entrance by using the Dirac  $\delta$  function represent the residence time distribution of the tested phase in apparatus. The use of the above relationships for the determination of  $Pe$  enables us to derive some correlation equations for the calculation of the Péclet numbers by means of the quantities characterizing the flow in extractor. These relationships were published mainly for continuous phase. Since the dispersed phase may be regarded as the second continuous phase with respect to its properties (which is valid only approximately), in literature it is recommended to use the above relations in equal form for that phase, too. It applies to the relations according to Strand [2]

$$\frac{n}{Pe_d} Z = 0.5 + 0.09 Z \frac{D_r N}{U_d} \left( \frac{D_r}{D_c} \right)^2 \left( \frac{D_s^2 - D_r^2}{D_c^2} \right) \quad (13)$$

Miyauchi [5]

$$\frac{n}{Pe_d} = 0.5 + 0.017 Z \frac{D_r N}{U_d n} N_p^{1/3} \left( \frac{D_c}{h_m} \right)^{1/2} \quad (14)$$

and Mišek [6]

$$\frac{n}{Pe_d} = 0.5 + k_0 Z \frac{D_r N}{U_d n} \left( \frac{N_p D_r^5 D_s^6}{D_c^8 h_m} \right)^{1/3} \quad (15)$$

where  $k_0 = 15.77 \text{ cm}^{1/3}$  and

$$N_p = \frac{P_d}{N^3 D_r^5 Q_c} = 0.069 \left( \frac{Q_c D_r^2 N}{\mu_c} \right)^{-0.155} \quad (16)$$

The quantity  $Z$  is the hold-up defined as the ratio of the volume of the dispersed phase to the volume of the working part of apparatus.

Rod [7] investigated the magnitude of longitudinal mixing due to mechanic stirring in dispersed phase on eliminating the influence of different particle sizes on this process. The author used solid particles of spherical shape in his experiments and found out a correlation relation. After rearranging this relation may be expressed in the form

$$\frac{\varepsilon_d}{U_d h_m} = 0.7 + 0.02 c_R Z \frac{D_r N}{U_d} \quad (17)$$

where  $c_R$  is the constriction factor given by the least value of the ratio of some internal free cross-section to the cross-section of column.

### Theoretical

The model idea of the mechanism of longitudinal mixing in dispersed phase is founded on the basic model of longitudinal mixing which assumes the additive effect of two factors determining this phenomenon. For a dispersed phase it may be supposed that the character of its flow is determined by the resulting effect of mechanical mixing and the distribution of fall velocities of drops owing to their different sizes. Hence, it holds according to [1]

$$\frac{1}{Pe_d} = K_0 + \left[ \frac{1}{Pe} \right]_{\text{mix}} + \left[ \frac{1}{Pe} \right]_{\text{vel}} \quad (18)$$

The contribution of mixing  $[1/Pe]_{\text{mix}}$  may be expressed by means of the procedure used for continuous phase [1], *i.e.* of the equation

$$\left[ \frac{1}{Pe} \right]_{\text{mix}} = K_1 \frac{D_r N Z}{U_d n} \quad (19)$$

The determination of the second contribution may be founded on the idea that the motion of dispersed phase is divided into two steps. In the first step, the phase moves in a piston manner as far as the entrance into apparatus, the velocities of all drops are equal and the dispersion of their residence times is equal to zero. In the second step, the proper flow through apparatus with a constant distribution of

residence times and their dispersion takes place. Thus according to [4], the dispersions of residence times at the entrance into apparatus  $\sigma_1^2$  and at its end  $\sigma_2^2$  may be regarded as responses of a certain input signal and since  $\sigma_1^2 = 0$ , we obtain according to (12)

$$\frac{2}{Pe} = \sigma_2^2(\vartheta) \quad (20)$$

where  $Pe$  is the apparent Péclet number characterizing the motion of dispersed phase with respect to the distribution of drop sizes and may be, therefore, regarded as  $Pe_{vel}$ . With respect to the dimensionless time used, the dispersion  $\sigma_2^2(\vartheta)$  is defined by the following expression

$$\sigma_2^2(\vartheta) = \left( \frac{U_d}{h_c Z} \right)^2 \left[ \int_0^\infty t^2 k(t) dt - \bar{t}^2 \right] = \left( \frac{U_d}{h_c Z} \right)^2 \sigma^2(t) \quad (21)$$

The mean residence time  $\bar{t}$  is given by the equation

$$\bar{t} = \int_0^\infty t k(t) dt \quad (22)$$

where  $k(t)$  represents the residence time distribution and  $h_c Z/U_d$  is the mean residence time of the dispersed phase in apparatus. The dispersion of the residence times of drops is, however, influenced by their aggregation and redispersion and according to the theory of aggregation height [8] the following relation between the dispersion of the residence times without aggregation  $\sigma_a^2$  and with aggregation  $\sigma_b^2$  is valid

$$\sigma_a^2 = n_z \sigma_b^2 \quad (23)$$

where  $n_z$  is the number of drop aggregations in an extractor of the height  $h_c$  while

$$h_c = n_z H_c \quad (24)$$

The quantity  $H_c$  is the so-called height of aggregation which may be calculated from the equation

$$H_c = \frac{1}{C} \frac{h_c}{Z \bar{t}} \sqrt{\frac{v}{E'}} \quad (25)$$

The value of the constant  $C$  in this equation depends on the circumstance whether the aggregation is rapid, *i.e.* every aggregation brings about coalescence, or slow. In the first case  $C = 4.2 \times 10^{-3}$  and in the second one  $C = 4.2 \times 10^{-5}$ . By combining eqns (20), (21), and (23), we obtain

$$\left[ \frac{2}{Pe} \right]_{vel} = \frac{U_d^2 \sigma^2(t)}{n_z h_c^2 Z^2} \quad (26)$$

Because of the difference between the real behaviour of the dispersed phase and the above assumptions, it is more convenient to write this relationship in the form

$$\left[ \frac{1}{Pe} \right]_{\text{vel}} = K_2 \left( \frac{U_d^2 \sigma^2(t)}{n_z h_c^2 Z^2} \right)^{K_3} \quad (27)$$

By inserting (19) and (27) into (18) we obtain the final expression for  $Pe_d$

$$\frac{1}{Pe_d} = K_0 + K_1 \frac{N D_r Z}{U_d n} + K_2 \left( \frac{U_d^2 \sigma^2(t)}{n_z h_c^2 Z^2} \right)^{K_3} \quad (28)$$

The values of the constants  $K_0$ — $K_3$  must be determined experimentally.

### Experimental

The experiments aimed at determining the coefficients of longitudinal mixing in both phases from the concentration profiles in the system water—acetone—toluene (depleted dispersed phase) were carried out in a laboratory rotary-disc extraction column the description of which as well as the physical properties of the system and the methods of the determination of profile had been the topic of previous papers [1, 9]. Besides, the values of  $\epsilon_c$  in continuous phase were indirectly measured from the distribution of the residence times of drops at a certain point of column by using a signal at the entrance in the form of the Dirac  $\delta$  function simulated by a rapid feed of a concentrated KCl solution (0.5—1.5 ml) into the continuous phase immediately under interface. The distribution of residence times was determined from the time dependence of the concentration of KCl at the point where  $z$  was equal to 0.688. This concentration was measured by means of the conductance of the continuous phase. The equipment used consisted of a conductance cell, conductometer OK 102/1, voltage divider (in proportion 40:1), and recorder EZ 4. These conductance measurements were repeated 5—8 times for each experiment and from the values of  $Pe_c$  found the arithmetic mean was calculated.

Furthermore, some experiments were performed for the purpose of determining the distribution of residence times of the dispersed phase. These experiments were based on the impulse feed of a concentrated solution of the Sudan III dye at the entrance of this phase and the colorimetric investigation of the time dependence of colour intensity at another point of the column ( $z = 0.688$ ) which was recorded. Assuming the colour intensity of the phase is proportional to the concentration of the tracing substance, the curves thus obtained may be considered to represent the distribution of residence times and therefore they may be used for determining the values of  $Pe$ . There were merely a few experiments of this kind because toluene always had to be deprived of the tracing substance. The results obtained were mainly used for estimating the suitability of this method for dispersed phase.

The interfacial tension in the system water—toluene at ordinary experimental temperature and mean concentration of acetone in both phases was determined by the approximate separation method and was found to be 0.025 N/m.

## Results and discussion

By using eqns (1—5) and the equilibrium relationship, the values of  $Pe_d$  were calculated from the experimental concentration profiles for individual experiments. These values as well as the values of hold-up are quoted in Table 1. The velocities of phases and the number of revolutions at these experiments are presented elsewhere [9]. Before proper determination of the constants in eqn (28), it was necessary to determine  $\sigma^2(t)$  by means of eqn (21) and  $n_z$  by means of eqns (24) and (25) for each experiment.

Table 1

Comparison between the measured values of  $Pe_d$  and the values calculated according to recommended correlation relations and between the values of  $Pe_c$  found from concentration profiles and by means of conductometric measurements

Experiment	Z	$Pe_d$ $Pe_x$	$n_z$	$Pe_d$ (13)	$Pe_d$ (17)	$Pe_d$ (15)	$Pe_d$ (14)	$Pe_c$ concentration profile	$Pe_c$ conductance measurement
1	0.095	9.6	0.348	6.5	4.6	55.2	56.3	8.4	12.8
2	0.114	11.7	9.515	7.5	5.3	51.4	52.6	12.3	11.8
3	0.129	14.5	0.736	8.2	5.7	47.6	48.9	15.0	10.9
4	0.168	17.8	1.082	9.9	6.9	41.2	42.6	19.1	13.5
5	0.232	21.8	1.905	12.0	8.3	33.2	34.6	32.0	12.1
6	0.098	5.8	0.352	6.8	4.8	56.7	57.8	9.8	11.0
7	0.104	5.2	0.419	7.1	5.0	54.4	55.6	15.2	14.1
8	0.138	7.3	0.711	8.8	6.2	48.0	49.3	18.7	9.6
9	0.171	9.3	1.287	10.2	7.1	42.5	43.9	20.3	10.8
10	0.219	12.4	2.067	11.8	8.2	35.7	37.1	18.9	11.3
11	0.230	13.8	2.306	12.0	8.3	34.0	35.3	22.9	6.0
12	0.116	5.2	0.449	8.0	5.6	56.2	57.3	8.5	8.6
13	0.125	6.4	0.579	8.4	5.9	53.2	54.4	11.6	11.0
14	0.161	9.6	0.930	10.2	7.1	47.2	48.5	14.4	6.0
15	0.172	10.0	1.298	10.7	7.4	45.2	46.6	16.4	7.8
16	0.216	12.7	2.340	12.3	8.5	39.3	40.6	21.2	7.9
17	0.250	13.9	3.100	13.2	9.1	34.7	36.1	31.8	8.5
18	0.128	6.6	0.502	8.8	6.2	55.4	56.6	10.1	7.8
19	0.143	6.2	0.722	9.6	6.7	52.5	53.7	12.8	6.4
20	0.163	9.9	0.984	10.5	7.3	48.8	50.1	17.0	8.2
21	0.186	8.6	1.300	11.4	8.0	44.9	46.2	57.0	12.6
22	0.225	15.1	2.317	12.9	8.9	39.5	41.0	25.2	9.0
23	0.260	18.2	—	13.7	9.5	34.8	36.2	23.0	16.1
24	0.117	5.7	0.420	8.2	5.8	58.0	59.1	8.3	9.6
25	0.154	9.8	0.764	10.3	7.2	52.3	53.5	11.4	8.4
26	0.171	12.3	0.922	11.0	7.7	48.9	50.2	13.4	6.1
27	0.188	13.9	1.321	11.7	8.2	45.9	47.2	18.1	6.9



The equation for the calculation of hold-up at a given value of  $N$  put forward by Mišek [10]

$$U_M = \frac{U_d}{Z} + \frac{U_c}{1-Z} = \kappa u_0(1-Z) \exp \left[ Z \left( \frac{\bar{z}}{\bar{\alpha}} - 4.1 \right) \right] \quad (29)$$

makes evident that  $\ln U_M/(1-Z)$  depends linearly on  $Z$ . The slope of this relationship is  $(\bar{z}/\bar{\alpha} - 4.1)$  and the section on the axis  $y$  is  $\ln \kappa u_0$ . The symbol  $\bar{z}$  stands for the so-called coefficient of aggregation characterizing the ratio of drop sizes in the absence and in the presence of coalescence. It may be calculated for a rapid aggregation by means of equation [10]

$$\bar{z} = 1.59 \times 10^{-2} \left[ \frac{D_c}{v_c} \left( \frac{\bar{\sigma}}{d \rho_c} \right)^{0.5} \right]^{0.5} \quad (30)$$

The quantity  $\bar{\alpha}$  is called velocity exponent and occurs in the relationships between fall velocities and drop diameter of the type

$$u^\alpha = k d \quad (31)$$

and is a function of  $Re'$  the graphical form of which is also given in [10].

From the measured values of hold-up processed in the sense of eqn (29), the values of  $u_0$ ,  $Re'$ ,  $\bar{\alpha}$ ,  $\bar{z}$ , and  $\bar{z}_v$  have been obtained. These values are listed in Table 2. The value of  $d_{32}$  is the arithmetic mean of all values obtained in individual experiments carried out at a given  $N$  and  $\bar{z}_v$  is the value of the coefficient of aggregation calculated according to eqn (30).

It ensues from the values of  $\bar{\alpha}$  that the experiments were carried out in the region of validity of the Allen equation which relates the fall velocity of drop with its diameter [11]

$$u = 0.153 \frac{d^{1.14} \Delta \rho^{0.71} g^{0.71}}{\rho_c^{0.29} \mu_c^{0.43}} \doteq k_2 d \quad (32)$$

Table 2

Experimental values of the mean fall velocity of drops  $u_0$ ,  $Re'$ , and velocity exponent  $\bar{\alpha}$  and comparison between the experimental values of the coefficient of aggregation  $\bar{z}$  and the calculated ones  $\bar{z}_v$

$u_0$ m s <sup>-1</sup>	$Re'$	$\bar{\alpha}$	$\bar{z}$	$\bar{z}_v$
0.0606	179	1.08	4.55	2.40
0.0556	149	1.07	3.10	2.47
0.0395	93.8	1.02	4.55	2.53
0.0206	43.5	0.93	7.05	2.62
0.1021	181	1.09	9.91	2.72
0.0332	23.5	0.86	3.61	2.78

The values of the coefficient of aggregation indicate a regime of rapid aggregation, which determines the value of the constant  $C$  in eqn (25). The values of  $n_z$  thus calculated are given in Table 1.

Assuming the distribution of drop sizes is governed by the Schwarz relation [9] and the fall velocity of drops by eqn (32), the mean values of the distribution of drop residence times  $\bar{t}$  in apparatus may be calculated on the basis of the procedure described in [1] by means of the equation

$$\bar{t} = \frac{24h_c}{k_2 \bar{a} G} \left( 1 + \frac{\bar{a}}{d_m} + \frac{\bar{a}^2}{2d_m^2} + \frac{\bar{a}^3}{6d_m^3} + \frac{\bar{a}^4}{24d_m^4} \right) \quad (33)$$

where

$$G = 6 \left( 1 + \frac{\bar{a}}{d_m} + \frac{\bar{a}^2}{2d_m^2} + \frac{\bar{a}^3}{6d_m^3} \right) \quad (34)$$

For the dispersion of the distribution of residence times  $\sigma^2(t)$ , it holds

$$\sigma^2(t) = \frac{120h_c^2}{k_2^2 \bar{a}^2 G} \left( 1 + \frac{\bar{a}}{d_m} + \frac{\bar{a}^2}{2d_m^2} + \frac{\bar{a}^3}{6d_m^3} + \frac{\bar{a}^4}{24d_m^4} + \frac{\bar{a}^5}{120d_m^5} \right) \quad (35)$$

By using this equation, the known values of  $n_z$ , and pertinent quantities, the numerical values of the expressions in parentheses of eqn (28) may be calculated for each experiment while the constants  $K_0$ — $K_3$  may be determined from the measured values of  $Pe_d$  by the method of least squares. Thus we obtain

$$\frac{1}{Pe_d} = 7.72 \times 10^{-3} + 2.64 \times 10^{-2} \frac{D_r N Z}{U_d n} + 2.65 \left( \frac{\sigma^2(t) U_d^2}{n_z h_c^2 Z^2} \right)^{0.75} \quad (36)$$

$$\delta = 0.1715, \quad \delta_m = 0.3968.$$

The quantities  $\delta$  and  $\delta_m$  represent the mean error and the maximum relative error, respectively, and are defined in more detail in [9]. A confrontation of the measured values of  $Pe_d$  with the above equation is shown in Fig. 5.

On the basis of dimensional analysis as well as the general form of criterion function adapted to the experimental conditions [9], the following relationship between  $Pe_d$  and pertinent quantities has been ascertained

$$Pe_d = 1.83 \times 10^{33} \alpha^{-2.82} A^{-3.66} Re^{-7} \quad (37)$$

$$\delta = 0.1840, \quad \delta_m = 0.4427.$$

Eqns (36) and (37) may be considered valid in the range of above-mentioned errors merely in the region of the conditions for which they have been derived and for this reason, the general validity of the model idea leading to eqn (36) cannot be, for the time being, appraised. The influence of geometrical arrangement of the

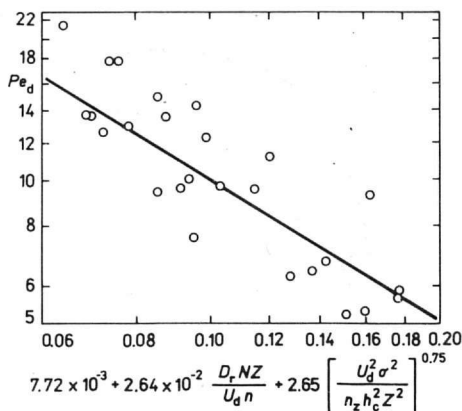


Fig. 5. Confrontation of the measured values of  $Pe_d$  with expression (36).

intrinsic construction elements of extractor and of physical properties on the values of the constants  $K_0$ — $K_3$  should not be, however, significant according this idea whereas it should manifest itself mainly in the distribution of drop sizes.

Furthermore, the values of  $Pe_d$  were compared with those calculated according to correlation relations (13—15) and (17) the values of which are given in Table 1. It ensues from this comparison that the Strand relation (13) is the most convenient under experimental conditions used in spite of that it was originally derived for continuous phase for which it was, however, found to be unsatisfactory [1].

As obvious from Table 1, the value of  $\varepsilon_d$  decreases with increasing number of revolutions because the number of drops of the dispersed phase increases (and the number of aggregations, too), their diameter decreases and the distribution of sizes approaches a monodispersed system, which results in a decrease in the value of  $\sigma^2(t)$ . Consequently, the term  $[1/Pe]_{vel}$  in eqn (28) decreases while the term  $[1/Pe]_{mix}$  increases simultaneously. On the basis of this fact it may be assumed that the effect of the distribution of drop sizes on flow character prevails over the effect of mechanical mixing in the region of the experiments performed similarly as in continuous phase [1].

This study also includes the indirect determination of the coefficient of longitudinal mixing in continuous phase by means of nonstationary analysis. This determination has been based on the assumption that the time coordinate of the maximum  $\vartheta_{max}$  of the residence time distribution of the tracing substance in the sense of eqn (8) is a function of  $Pe$  the form of which may be found by determining  $\vartheta_{max}$  from the curves (8) for different values of  $Pe$  (Fig. 6). Then the following inequality is valid

$$0 \leq \vartheta_{max} \leq 1 \quad (38)$$

This relation may be applied to the determination of the value of  $Pe$  from the distribution of residence times of the tracing substance as found experimentally.

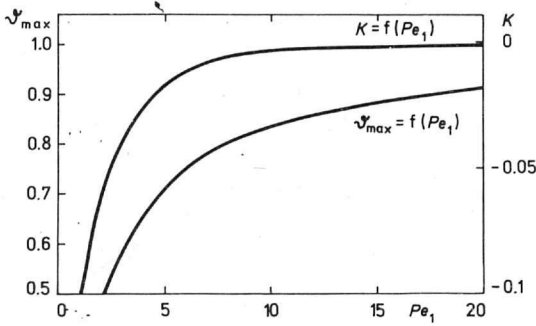


Fig. 6. Variation of  $\vartheta_{max}$  and  $K$  with the Péclet number referred to bounded model ( $Pe_1$ ).

This procedure based on calculating dispersions from the experimental curves and using relation (9) or (11) has shortcomings consisting in that the accuracy of determining  $\sigma^2(\vartheta)$  is relatively low.

Since relation (8) is valid for half-infinite model and the laboratory apparatus approximately fulfilled the conditions of bounded model, it is necessary to find out the error due to the exchange of these models. A certain value of dispersion  $\sigma^2(\vartheta)$  given by the relation valid for half-infinite model (9) which would be really found at a given point if the apparatus were such a model corresponds to the value of  $Pe_1$  found by means of  $\vartheta_{max}$ . According to eqn (11), another value of  $Pe_2$ , however, corresponds to that value of dispersion in bounded model. By using the last two equations cited, the dependence on  $Pe_1$  of the correction factor  $K$  defined by the equation

$$K = \frac{Pe_2 - Pe_1}{Pe_2} \quad (39)$$

may be determined for different positions of the measuring place in the apparatus, in our case for  $a' = 0.312$ . This relationship is also depicted in Fig. 6 and it follows from it that  $K$  shows a very low value which is smaller than the error of measurement in the range of medium and higher values of  $Pe$ . It means that  $a'$  represents a sufficiently large distance from the edge of apparatus for both models to give almost equal results.

The values  $Pe_c$  found by the above procedure under working conditions of individual experiments are given in Table 1. A comparison between the values of  $Pe_c$  determined from concentration profiles [1] and those calculated from correlation relations shows that even an approximate agreement does not occur in any case and it seems that there is no relation between the values of  $Pe_c$  thus determined and pertinent quantities. It may be, to a certain extent, explained by that the errors due to the presence of the second phase and probably to the substance transfer, too, were significantly effective.

The application of this method to dispersed phase has shown that the properties of this phase may not be considered similar to the properties of continuous phase just because of the discrete character of its particles. Toluene containing the tracing substance was gradually dispersed in a column. First of all, large drops proceeding more rapidly were formed. These drops gradually split into smaller and slower drops which occasionally collided with noncoloured drops and dispersed again so that the concentration of the tracing substance in them decreased. If we take into consideration that one intensely coloured drop absorbs as much light as several less coloured equal drops, then we see that such a measurement cannot give an image of the residence time distribution of the tracing substance in a dispersed phase. It is also evidenced by the fact that the time  $\vartheta_{\max}$  was always approximately equal to one or greater, which indicated that there was only a slight longitudinal mixing in the phase though the direct measurements led to other conclusions. The cases in which  $\vartheta_{\max} > 1$  are without any physical significance. The situation in the continuous phase is different. The particles of the tracing substance freely spread in the whole volume according to real hydrodynamic conditions while such spreading in a dispersed phase is limited only to the splitting and coalescence of drops. Besides, some subjective shortcomings of that method occurred. These shortcomings consisted in incidentally catching or retarding some coloured drops in the internal construction of column. Then these drops passed along the photocell of colorimeter later and distorted the results of measurement.

On the basis of the experience obtained from the use of nonstationary analysis for the investigation of the character of two-phase flow by means of the conductometric and colorimetric indication of the tracing substance, it may be stated that this method is not suited (especially for dispersed phase) and it is more convenient to determine  $Pe$  from the concentration profiles of the transferred substance. However, this procedure has also its limitations, as described in [1].

### Symbols

$a$	specific interfacial surface
$a'$	dimensionless distance (Fig. 3)
$\bar{a}$	parameter for the drop size distribution according to Schwarz
$A = \frac{D_r N Z}{U_d}$	quantity in eqn (37)
$c$	concentration of tracing substance
$c_R$	constriction factor in eqn (17)
$d$	drop diameter
$d_m$	maximum drop diameter
$d_{32}$	mean drop diameter of the order 3,2

$D_r$	rotor diameter
$D_s$	stator diameter
$D_c$	column diameter
$E'$	power input for a unit of mixed mass
$G$	quantity defined by eqn (34)
$h$	distance
$h_m$	height of stirring stage
$h_c$	length of column
$H_c$	height of aggregation
$k, K$	constants, correction factor defined by eqn (39)
$k(t)$	residence time distribution of drops in column
$K_x$	overall coefficient of mass transfer
$L$	length
$n$	number of stirring stages
$n_z$	number of effective aggregations
$N$	number of rotor revolutions per time unit
$No = \frac{K a h_c}{U}$	number of transfer units of apparatus
$N_p$	input criterion defined by expression (16)
$P_d$	power input per one column disc
$Pe = \frac{U h_c}{\varepsilon}$	Péclet number
$Re = \frac{h_c Q_c U}{Z \mu_c}$	Reynolds number from eqn (37)
$Re' = \frac{d_{32} u_0 Q_c}{\mu_c}$	Reynolds number in Table 2
$t$	time
$\bar{t}$	mean value defined by eqn (22)
$u$	fall velocity of drop
$u_0$	mean fall velocity of drops
$U$	velocity of phase
$U_M$	quantity defined by eqn (29)
$x$	concentration of depleted phase
$X$	concentration defined by eqn (3)
$y$	concentration of enriched phase
$Y$	concentration defined by eqn (5)
$z = \frac{h}{h_c}; z = \frac{h}{L}$	dimensionless distance
$\bar{z}$	coefficient of aggregation
$Z$	hold-up of dispersed phase
$\alpha = \frac{U_d(1-Z)}{U_c Z}$	quantity in eqn (37)

$\bar{\alpha}$	velocity exponent
$\delta$	relative error
$\varepsilon$	coefficient of longitudinal mixing
$\mu_c$	dynamical viscosity
$\kappa = \left(\frac{D_s}{D_c}\right)^2$	relative free cross-section of column
$\nu$	kinematic viscosity
$\sigma^2$	dispersion of distribution
$\rho$	phase density
$\Delta\rho$	difference between densities of continuous and dispersed phase
$\theta$	dimensionless time (relation (10))
$\bar{\sigma}$	interfacial tension

### Indices

c	continuous phase
d	dispersed phase
E	exit of enriched phase
F	entrance of depleted phase from outside
H	entrance of enriched phase from inside
O	entrance of depleted phase from inside
R	exit of depleted phase
S	entrance of enriched phase from outside
x	depleted phase
y	enriched phase
z	reverse flow, value in the distance z from the entrance of depleted phase

### Exponents

\* equilibrium state

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