Control of Biotechnological Complex Systems

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Applications of hierarchical (or multilevel) technique for solution of some optimized control problems in fermentation systems are described. Three approaches to control of biotechnological complex systems are introduced. Results presented show some advantages of the method of hierarchical control in comparison to some traditional one-level optimization methods. In solving this problem two control schemes are used. In the first control scheme the method of objective coordination is used. In the second one the prediction method of coordination is used, with the aim to minimize selected disturbances in state and control variables system.

In the last years an emphasis has been laid on applicability of optimal control in commercial fermentation. In the paper a possibility and usefulness of application of hierarchical optimization technique in D-gluconic acid fermentation is outlined. The process of D-gluconic acid production was chosen, because it utilizes the most of biochemical processes which fermentation processes usually consist of. Besides this D-gluconic acid is also commonly used in food-stuff production, in pharmacy, and in other industries.

In the paper a nonlinear model of this process is introduced. Objective function is maximized for certain dosage of fermentation broth with respect to concentration of D-gluconic acid.

Three control schemes will be introduced. In the first scheme one-level optimization technique on the basis of conjugate gradient method is used. In the second one a hierarchical approach is realized using the aim coordination method. In the third scheme the prediction method applied to minimization of any deviation in state and command variables from the desired level which follows after disturbance in any or in all states is used.

Process Description

Conversion of D-glucose to D-gluconic acid is a simple oxidation of aldehydic group to carboxyl group. This conversion may be performed by help of some specific microorganisms.

The whole reaction mechanism can be described as follows.

Cell growth

$$\text{D-glucose} + \text{cells} \rightarrow \text{cells} \qquad (A)$$

D-Glucose oxidation

$$\text{D-glucose} + O_2 \xrightarrow[\text{oxidase}]{\text{D-glucose}} \text{D-gluconolactone} + H_2O \qquad (B)$$

D-Gluconolactone hydrolysis

$$\text{D-gluconolactone} + H_2O \rightarrow \text{D-gluconic acid} \qquad (C)$$

H_2O_2 decomposition

$$H_2O_2 \xrightarrow{\text{catalysis}} H_2O + 1/2\ O_2 \qquad (D)$$

As can be seen from these equations, D-gluconic acid is produced by the enzymatic oxidation of D-glucose to D-gluconolactone and D-gluconolactone hydrolysis. The by-product of the first reaction, hydrogen peroxide, is decomposed to water and oxygen during enzyme catalysis.

Dynamic mathematical model of this process is of the form published in [1]

$$\dot{x}_1 = b_1 x_1 \left[1 - \frac{x_1}{b_2}\right] \qquad (1)$$

$$\dot{x}_2 = \frac{b_3 x_1 x_4}{b_4 + x_4} - 0.9082 b_5 x_2 \qquad (2)$$

$$\dot{x}_3 = b_5 x_2 \tag{3}$$

$$\dot{x}_4 = -1.011 \left[\frac{b_3 x_1 x_4}{b_4 + x_4}\right] \tag{4}$$

with initial conditions

$$x_1(0) = 0.5 \text{ OD cm}^{-3} \quad x_2(0) = 0.0 \text{ mg cm}^{-3}$$
$$x_3(0) = 0.0 \text{ mg cm}^{-3} \quad x_4(0) = 50 \text{ mg cm}^{-3} \tag{5}$$

where x_1 is the cell optical density (OD cm^{-3}), x_2 D-gluconolactone concentration (mg cm^{-3}), x_3 D-gluconic acid concentration (mg cm^{-3}), x_4 D-glucose concentration (mg cm^{-3}). b_1 (h^{-1}), b_2 (OD cm^{-3}), b_3 (mg OD^{-1} h^{-1}), b_4 (mg cm^{-3}), and b_5 (h^{-1}) are constants, and dots denote a time derivative of the respective variables.

Limitations on quantities θ and pH are following

$$25 \leq \theta \leq 35.4 \tag{6}$$

$$5.4 \leq \text{pH} \leq 7.0 \tag{7}$$

where θ is temperature/°C.

Solution of the problem is based on the linearized system model.

The linearized system description is as follows from [1].

$$\dot{x}_1 = -1.05x_1 - 0.01u_1 - 0.4u_2 \tag{8}$$

$$\dot{x}_2 = 0.003x_1 - 6.7x_2 + 1.7x_4 - 6 \times 10^{-4}u_1 - 8 \times 10^{-3}u_2 \tag{9}$$

$$\dot{x}_3 = 7.4x_2 + 8 \times 10^{-4}u_1 + 0.02u_2 \tag{10}$$

$$\dot{x}_4 = -0.003x_1 - 1.7x_4 - 1.1 \times 10^{-4}u_1 - 2.5 \times 10^{-3}u_2 \tag{11}$$

Optimization Methods

The objective of this study is to obtain 99.9 % of maximum theoretical yield of D-gluconic acid during 8 h, then to minimize any disturbances of state and of command variables which start to influence the fermentation process in the 8th hour. The aim of this procedure is to determine the control vector $\boldsymbol{u} \in \boldsymbol{U}$, which maximizes the objective function

$$J = x_3(t_f) \qquad t_f = 8 \text{ h} \tag{12}$$

with respect to constrains (6) and (7), where t_f is the time, after which disturbances start to influence the system.

The necessary optimal control conditions were obtained from the principle of maximum.

First scheme

In this scheme optimum values for problem solution are obtained using the conjugate gradient method.

Second scheme

In this scheme the optimum values are obtained by decomposition of the whole system into subsystems with the aim to decrease the claim on memory capacity of a micro-processor system. This approach is the so-called hierarchical one or multilevel technique. These subsystems are coordinated by the help of a coordinator. The system is decomposed into two subsystems. In Fig. 1 a scheme of the subsystem coordination is depicted.

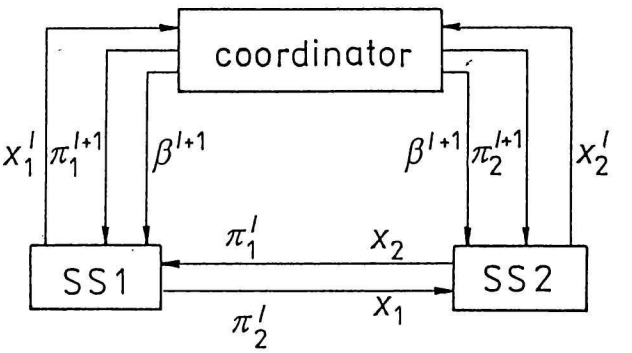

Fig. 1. Block diagram of the hierarchical control of complex (for D-gluconic acid production). SS1, SS2 – subsystems No. 1 and No. 2.

The first subsystem

Let us consider that the first subsystem is described by eqns (1) and (3) and the second one by eqns (2) and (4). Let the coupling quantities be as follows

$$\pi_1 = x_2 \quad \text{and} \quad \pi_2 = x_1 \tag{13}$$

Then the objective function (12) will be

$$I = x_3(t_f) + \int_0^{t_f} [\beta_1(\pi_1 - x_2) + \beta_2(\pi_2 - x_1)]\text{d}t \tag{14}$$

where $\boldsymbol{\beta} = [\beta_1, \beta_2]^{\text{T}}$ is the Lagrangian multiplicative vector and the Hamiltonian function is

$$H = \beta_1(\pi_1 - x_2) + \beta_2(\pi_2 - x_1) + p_1 \left[b_1 x_1 \left(1 - \frac{x_1}{b_2}\right)\right]$$
$$+ p_2 \left[b_3 \frac{\pi_2 x_4}{b_4 + x_4} - 0.9082 b_5 x_2\right] + p_3[b_5 \pi_1]$$
$$+ p_4 \left[-1.011 \left(b_3 \frac{\pi_2 x_4}{b_4 + x_4}\right)\right] \tag{15}$$

For the first subsystem there will be maximized

$$I = x_3(t_f) + \int_0^{t_f} (\beta_1\pi_1 - \beta_2 x_1)\mathrm{d}t \qquad (16)$$

subject to

$$\dot{x}_1 = b_1 x_1 \left(1 - \frac{x_1}{b_2}\right) \qquad \dot{x}_3 = b_5\pi_1 \qquad (17)$$

and with initial conditions

$$x_1(0) = 0.5 \qquad x_3(0) = 0.0 \text{ mg cm}^{-3} \qquad (18)$$

Then the Hamiltonian function for the first subsystem is

$$H_1 = \beta_1\pi_1 - \beta_2 x_1 + p_1\left[b_1 x_1\left(1 - \frac{x_1}{b_2}\right)\right] + p_3(b_5\pi_1) \qquad (19)$$

For the second subsystem there will be maximized

$$I_2 = \int_0^{t_f} (\beta_2\pi_2 - \beta_1 x_2)\mathrm{d}t \qquad (20)$$

subject to

$$\dot{x}_2 = b_3\frac{\pi_2 x_4}{b_4 + x_4} - 0.9082 b_5 x_2 \qquad (21)$$

$$\dot{x}_4 = -1.0011 b_3\left[\frac{\pi_2 x_4}{b_4 + x_4}\right] \qquad (22)$$

with initial parameters

$$x_2(0) = 0 \quad x_4(0) = 50.0 \text{ mg cm}^{-3} \qquad (23)$$

Then the Hamiltonian function for the second subsystem is following

$$H_2 = \beta_2\pi_2 - \beta_1 x_2 + p_2\left[b_3\frac{\pi_2 x_4}{b_4 + x_4} - 0.9082 b_5 x_2\right] + p_4\left[-1.011 b_3\frac{\pi_2 x_4}{b_4 + x_4}\right] \qquad (24)$$

For the coordination the predictive principle is used. In this principle the global control term obtains the values for $\boldsymbol{\beta}(t)$ and $\boldsymbol{\pi}(t)$.

The method of data equality, see [2, 3], is utilized for data determination of coordination variables π_i and β_i.

This method manifests good convergence properties.

Coordination variables are as follows

$$\pi_1^{(I+1)} = x_2^{(I)} \qquad (25)$$

$$\pi_2^{(I+1)} = x_1^{(I)} \qquad (26)$$

$$\beta_1^{(I+1)} = -b_5^{(I)} p_3^{(I)} \qquad (27)$$

$$\beta_2^{(I+1)} = (1.011 p_4^{(I)} - p_2^{(I)})\frac{b_3^{(I)} x_4^{(I)}}{b_4^{(I)} + x_4^{(I)}} \qquad (28)$$

where $\boldsymbol{p}$ is the costate vector and $^{(I)}$ is the iteration index.

For the second level an error e was chosen as

$$e = \int_0^{t_f}\left[\sum_{i=1}^{l} |\pi_i^{(I+1)} - \pi_i^{(I)}|\right]\mathrm{d}t \qquad (29)$$

Hence a two-level optimization structure is considered. At the first level for given $\boldsymbol{\pi}$ and $\boldsymbol{\beta}$ two independent minimization problems are solved by using the conjugate gradient method. At the second level the $\boldsymbol{\pi}$ and $\boldsymbol{\beta}$ trajectories are utilized by using the interactions balance method to maximize the global objective function.

The third scheme utilizes the objective coordination method and predictive coordination method. Both these methods are described in the following two chapters.

APPLICATION OF OBJECTIVE COORDINATION METHOD

The aim is to reach optimal states by decomposition of the whole system into subsystems. In this way the required size of microcomputer memory is decreased. This method is called a hierarchical control method [4]. The whole system is decomposed into the two subsystems.

The problem solution is based on the linearized system model.

The coupling quantities are

$$\pi_1 = x_2 \qquad \pi_2 = x_1 \qquad (30)$$

The system block diagram is depicted in Fig. 1.

Functional has the form

$$J = x_3(t_f) + \int_0^{t_f}\{\beta_1(\pi_1 - x_2) + \beta_2(\pi_2 - x_1) + 0.5[(u_1 - 35.4)^2 + (u_2 - 7)^2]\}\mathrm{d}t \qquad (31)$$

where $\boldsymbol{\beta} = [\beta_1, \beta_2]^{\mathrm{T}}$ is the vector of Lagrangian multipliers.

The Hamiltonian function has the form

$$\begin{aligned} H = {} & \beta_1(\pi_1 - x_2) + \beta_2(\pi_2 - x_1) \\ & + 0.5[(u_1 - 35.4)^2 + (u_2 - 7)^2] \\ & + \lambda_1(-1.05x_1 - 0.01u_1 - 0.4u_2) \\ & + \lambda_2(0.003\pi_2 - 6.7x_2 + 1.7x_4 - 6\times 10^{-4}u_1 \\ & - 8\times 10^{-3}u_2) \\ & + \lambda_3(7.4\pi_1 + 8\times 10^{-4}u_1 + 0.02u_2) \\ & + \lambda_4(-0.003\pi_2 - 1.7x_4 - 1.1\times 10^{-4}u_1 \\ & - 2.5\times 10^{-3}u_2) \end{aligned} \qquad (32)$$

where $\lambda_1, \lambda_2, \ldots, \lambda_4$ are Lagrangian multipliers.

From optimality conditions [5] it follows that

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = \beta_2 + 1.05\lambda_1 \quad (33)$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = \beta_1 + 6.7\lambda_2 \quad (34)$$

$$\dot{\lambda}_3 = -\frac{\partial H}{\partial x_3} = 0 \quad (35)$$

$$\dot{\lambda}_4 = -\frac{\partial H}{\partial x_4} = -1.7\lambda_2 + 1.7\lambda_4 \quad (36)$$

$$\frac{\partial H}{\partial \lambda_1} = \dot{x}_1 \; \frac{\partial H}{\partial \lambda_2} = \dot{x}_2 \; \frac{\partial H}{\partial \lambda_3} = \dot{x}_3 \; \frac{\partial H}{\partial \lambda_4} = \dot{x}_4 \quad (37)$$

$$\frac{\partial H}{\partial u_1} = u_1 - 35.4 - 0.01\lambda_1 - 6 \times 10^{-4}\lambda_2 + 8 \times 10^{-4}\lambda_3 - 1.1 \times 10^{-4}\lambda_4 = 0 \quad (38)$$

$$\frac{\partial H}{\partial u_2} = u_2 - 7.0 - 0.4\lambda_1 - 8 \times 10^{-3}\lambda_2 + 0.02\lambda_3 - 2.5 \times 10^{-3}\lambda_4 = 0 \quad (39)$$

From eqns (*38*) and (*39*) following relations for command variables u_1, u_2 could be obtained

$$u_1 = 35.4 + 0.01\lambda_1 + 6 \times 10^{-4}\lambda_2 - 8 \times 10^{-4}\lambda_3 + 1.1 \times 10^{-4}\lambda_4 \quad (40)$$

$$u_2 = 7.0 + 0.4\lambda_1 + 8 \times 10^{-3}\lambda_2 - 0.02\lambda_3 - 2.5 \times 10^{-3}\lambda_4 \quad (41)$$

We decompose this system into two subsystems. The first subsystem is described by the relations (*8*) and (*10*) with following initial conditions

$$x_1(0) = 0.5 \text{ OD cm}^{-3} \quad x_3(0) = 0.0 \text{ mg cm}^{-3} \quad (42)$$

For this subsystem the corresponding functional has the form

$$\max J_1 = x_3(t_f) + \int_0^{t_f} \{\beta_1\pi_1 - \beta_2 x_1 + 0.5[(u_1 - 35.4)^2 + (u_2 - 7.0)^2]\}\mathrm{d}t \quad (43)$$

The Hamiltonian function will be

$$H_1 = \beta_1\pi_1 - \beta_2 x_1 + 0.5[(u_1 - 35.4)^2 + (u_2 - 7.0)^2] + \lambda_1(-1.05x_1 - 0.01u_1 - 0.4u_2) + \lambda_3(7.4\pi_1 + 8 \times 10^{-4}u_1 + 0.02u_2) \quad (44)$$

Its derivative is

$$\frac{\partial H_1}{\partial \pi_1} = \beta_1 + 7.4\lambda_3 = 0$$

Hence it follows that

$$\beta_1 = -7.4\lambda_3 \quad (45)$$

The second subsystem is described by relations (*9*) and (*11*) and by following initial conditions

$$x_2(0) = 0.0 \quad x_4(0) = 50.0 \text{ mg cm}^{-3}$$

For the second subsystem the corresponding functional is following

$$\max J_2 = \int_0^{t_f} \{\beta_2\pi_2 - \beta_1 x_2 + 0.5[(u_1 - 35.4)^2 + (u_2 - 7.0)^2]\}\mathrm{d}t \quad (46)$$

and the Hamiltonian function

$$H_2 = \beta_2\pi_2 - \beta_1 x_2 + 0.5[(u_1 - 35.4)^2 + (u_2 - 7.0)^2] + \lambda_2(0.003\pi_2 - 6.7x_2 + 1.7x_4 - 6 \times 10^{-4}u_1 - 8 \times 10^{-3}u_2) + \lambda_4(-0.003\pi_2 - 1.7x_4 - 1.1 \times 10^{-4}u_1 - 2.5 \times 10^{-3}u_2) \quad (47)$$

From the relation

$$\frac{\partial H_2}{\partial \pi_2} = \beta_2 + 0.003\lambda_2 - 0.003\lambda_4 = 0 \quad (48)$$

it follows that

$$\beta_2 = -0.003\lambda_2 + 0.003\lambda_4 \quad (49)$$

For computation of the course of the state variables x_1, x_2, x_3, x_4 and of command variables eqns (*8—11*), (*33—36*), (*40*), and (*41*) were used. The target coordination method utilizes the prediction principle for coordination. For the given $\boldsymbol{\pi}$ and $\boldsymbol{\beta}$ values the first level solves an optimization problem solving at the same time two independent minimization problems. At the second level the quantities $\boldsymbol{\pi}$ and $\boldsymbol{\beta}$ are further predicted. In this way their trajectories are improved, the functionals (*12*) and (*43*) are maximized in such a way that following relations are fulfilled

$$\pi_1^{(I+1)} = x_2^{(I)} \quad \pi_2^{(I+1)} = x_1^{(I)} \quad \beta_1^{(I+1)} = -7.4\lambda_3$$

$$\beta_2^{(I+1)} = -0.003\lambda_2^{(I)} + 0.003\lambda_4^{(I)} \quad (50)$$

where $^{(I)}$ is the iteration index.

Computational algorithm is depicted in Fig. 2. The second level determines the error according to the relation

$$e = \int_0^{t_f} \left[\sum_{i=1}^{2}(\pi_i^{(I+1)} - \pi_i^{(I)})\right] \mathrm{d}t \quad (51)$$

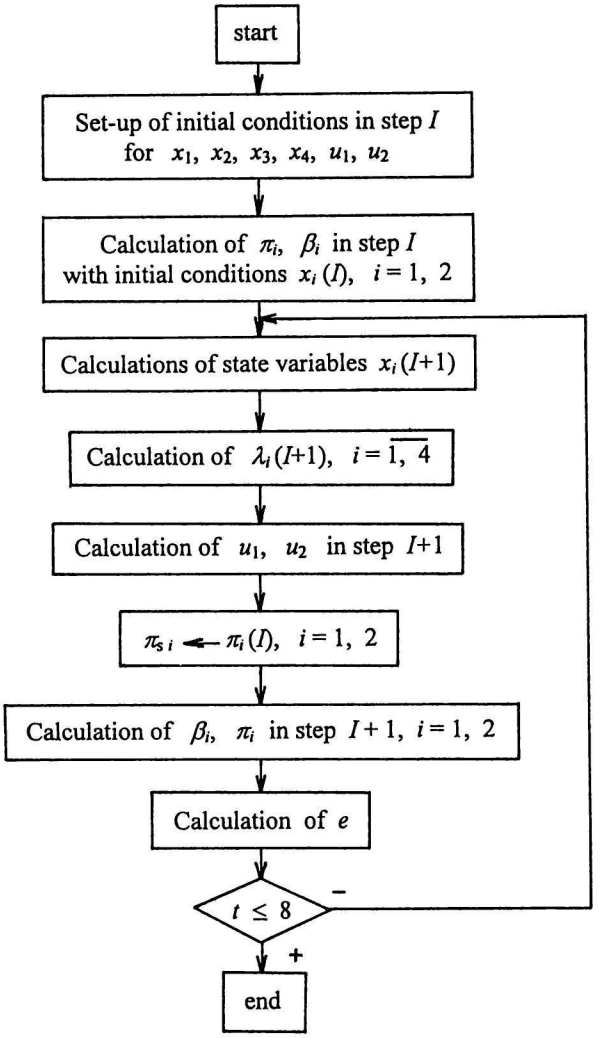

Fig. 2. Algorithm of the calculation according to objective coordination.

APPLICATION OF THE PREDICTION COORDINATION METHOD

In this part the disturbances in state and command variables are minimized. These started to manifest in the system after 8 h from the start of the fermentation process and they lasted for 1.5 h. The first level determines the course of the state and command variables. The second level predicts the state and command variables. Again decomposition of the system into two subsystems is here applied in this case in the form as described by eqns (*8—10*) and (*9—11*) with initial conditions

$$x_1(8) = -3.0030 \text{ OD cm}^{-3} \quad u_1(8) = 35.4\,^{\circ}\text{C}$$
$$x_2(8) = -0.0147 \text{ mg cm}^{-3} \quad u_2(8) = 7.0$$
$$x_3(8) = 55.7231 \text{ mg cm}^{-3}$$
$$x_4(8) = -0.0072 \text{ mg cm}^{-3}$$

Global functional is given as

$$J = 0.5 \int_0^{1.5} [(x_1 + 3.0030)^2 + (x_2 + 0.0147)^2 + (x_3 - 55.7231)^2 + (x_4 + 0.0072)^2 + (u_1 - 35.4)^2 + (u_2 - 7.0)^2]\mathrm{d}t \qquad (52)$$

Corresponding Hamiltonian function has the form

$$\begin{aligned} H = {} & \lambda_5[(x_1 + 3.0030)^2 + (x_2 + 0.0147)^2 \\ & + (x_3 - 55.7231)^2 + (x_4 + 0.0072)^2 \\ & + (u_1 - 35.4)^2 + (u_2 - 7.0)^2] \\ & + \lambda_1(-1.05x_1 - 0.01u_1 - 0.4u_2) \\ & + \lambda_2(0.003x_1 - 6.7x_2 + 1.7x_4 \\ & - 6 \times 10^{-4}u_1 - 8 \times 10^{-3}u_2) \\ & + \lambda_3(7.4x_2 + 8 \times 10^{-4}u_1 + 0.02u_2) \\ & + \lambda_4(-0.003x_1 - 1.7x_4 \\ & - 1.1 \times 10^{-4}u_1 - 2.5 \times 10^{-3}u_2) \end{aligned} \qquad (53)$$

From the principle of maximum it follows that

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial x_1} = 1.05\lambda_1 - 0.003\lambda_2 + 0.003\lambda_4 - 2x_1\lambda_5 - 6.006\lambda_5 \qquad (54a)$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial x_2} = 6.7\lambda_2 - 7.4\lambda_3 - 2x_2\lambda_5 - 0.0294\lambda_5 \qquad (54b)$$

$$\dot{\lambda}_3 = -\frac{\partial H}{\partial x_3} = -2x_3\lambda_5 + 11.4462\lambda_5 \qquad (54c)$$

$$\dot{\lambda}_4 = -\frac{\partial H}{\partial x_4} = -1.7\lambda_2 + 1.7\lambda_4 - 2\lambda_5 x_4 - 0.01446\lambda_5 \qquad (54d)$$

$$\dot{\lambda}_5 = -\frac{\partial H}{\partial x_5} = 0 \qquad (54e)$$

From the latter relation it follows that $\lambda_5(t_1) = 1$, where $t_1 = 1.5$ h.

From the above system of differential equations (*54*) the values of Lagrangian multipliers are determined.

From optimality conditions it can be further obtained

$$\frac{\partial H}{\partial u_1} = -0.01\lambda_1 - 6 \times 10^{-4}\lambda_2 + 8 \times 10^{-4}\lambda_3 - 1.1 \times 10^{-4}\lambda_4 + 2u_1\lambda_5 - 70.8\lambda_5 = 0$$

$$\frac{\partial H}{\partial u_2} = -0.4\lambda_1 - 8 \times 10^{-3}\lambda_2 + 0.02\lambda_3 - 2.5 \times 10^{-3}\lambda_4 + 2u_2\lambda_5 - 14\lambda_5 = 0$$

After some modifications the command variables u_1 and u_2 can be obtained in the following form

$$u_1 = (0.01\lambda_1 + 6 \times 10^{-4}\lambda_2 - 8 \times 10^{-4}\lambda_3 + 1.1 \times 10^{-4}\lambda_4 + 70.8\lambda_5)/2 \quad (55a)$$

$$u_2 = (0.4\lambda_1 + 8 \times 10^{-3}\lambda_2 - 0.02\lambda_3 + 2.5 \times 10^{-3}\lambda_4 + 14\lambda_5)/2 \quad (55b)$$

Let us form the deviations $\boldsymbol{y}$, $\boldsymbol{v}$ from the system equilibrium points $\boldsymbol{x}_e$, $\boldsymbol{u}_e$

$$\boldsymbol{y} = \boldsymbol{x} - \boldsymbol{x}_e \quad \boldsymbol{v} = \boldsymbol{u} - \boldsymbol{u}_e$$

where

$$\boldsymbol{x}_e = [-3.0030, -0.0147, 55.7231, -0.0072]$$

and

$$\boldsymbol{u}_e = [35.4, 7] \quad (56)$$

Then the system description by eqns (*8—11*) can be transformed to the deviation form

$$\dot{\boldsymbol{y}} = \mathbf{A}\boldsymbol{y} + \mathbf{B}\boldsymbol{v} + [\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) - \mathbf{A}\boldsymbol{y} - \mathbf{B}\boldsymbol{v}] = \mathbf{A}\boldsymbol{y} + \mathbf{B}\boldsymbol{v} + \boldsymbol{D} \quad (57)$$

where

$$\boldsymbol{D} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) - \mathbf{A}\boldsymbol{y} - \mathbf{B}\boldsymbol{v}$$

The following functional is to be minimized

$$J = 0.5 \int_0^{1.5} [\boldsymbol{y}^T \mathbf{Q} \boldsymbol{y} + \boldsymbol{v}^T \mathbf{R} \boldsymbol{v}] \mathrm{d}t \quad (58)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are unit matrices of the 4th order.

The Hamiltonian function is of the form

$$H = \frac{1}{2}\left[\boldsymbol{y}^T \mathbf{Q} \boldsymbol{y} + \frac{1}{2}\boldsymbol{v}^T \mathbf{R} \boldsymbol{v}\right] + \boldsymbol{p}[\mathbf{A}\boldsymbol{y} + \mathbf{B}\boldsymbol{v} + \mathbf{D}] + \boldsymbol{\pi}[\boldsymbol{y} - \boldsymbol{y}^\circ] + \boldsymbol{\beta}[\boldsymbol{v} - \boldsymbol{v}^\circ] \quad (59)$$

where $\boldsymbol{p}$ is the costate vector and $\boldsymbol{\pi}$ and $\boldsymbol{\beta}$ are the Lagrangian multipliers.

If $\boldsymbol{y} = \boldsymbol{y}^\circ$, $\boldsymbol{v} = \boldsymbol{v}^\circ$, then the equation describing the system has the following form

$$\dot{\boldsymbol{y}} = \mathbf{A}\boldsymbol{y} + \mathbf{B}\boldsymbol{v} + \boldsymbol{D}\Big|_{\substack{\boldsymbol{y}=\boldsymbol{y}^\circ \\ \boldsymbol{v}=\boldsymbol{v}^\circ}}$$

$$\boldsymbol{D}\Big|_{\substack{\boldsymbol{y}=\boldsymbol{y}^\circ \\ \boldsymbol{v}=\boldsymbol{v}^\circ}} = \boldsymbol{f}(\boldsymbol{x}_e + \boldsymbol{y}^\circ, \boldsymbol{u}_e + \boldsymbol{v}^\circ, t) - \mathbf{A}\boldsymbol{y}^\circ - \mathbf{B}\boldsymbol{v}^\circ \quad (60)$$

After splitting the system into two subsystems the state equations of the respective subsystems have the form as follows.

First subsystem

$$\dot{\boldsymbol{Y}}_1 = \mathbf{A}_1\boldsymbol{Y}_1 + \mathbf{B}_1\boldsymbol{V}_1 + \boldsymbol{D}_1^\circ \quad (61)$$

and the functional to be minimized is

$$J_1 = \int_0^{1.5} (\boldsymbol{Y}_1^T \mathbf{Q}_1 \boldsymbol{Y}_1 + \boldsymbol{V}_1^T \mathbf{R}_1 \boldsymbol{V}_1)\mathrm{d}t \quad (62)$$

where $\mathbf{Q}_1$ and $\mathbf{R}_1$ are unit matrices of the second order, and

$$\boldsymbol{Y}_1^T = [y_1, y_3] \quad \boldsymbol{V}_1^T = [v_1, v_2]$$

$$\mathbf{A}_1 = \begin{bmatrix} -1.05 & 0 \\ 0 & 0 \end{bmatrix} \quad \mathbf{B}_1 = \begin{bmatrix} -0.01 & 0 \\ 0 & 0.02 \end{bmatrix}$$

$$\boldsymbol{D}_1^\circ = \begin{bmatrix} f_1 \\ f_3 \end{bmatrix} - \mathbf{A}_1 \begin{bmatrix} y_1^\circ \\ y_3^\circ \end{bmatrix} - \mathbf{B}_1 \begin{bmatrix} v_1^\circ \\ v_2^\circ \end{bmatrix}$$

Hence

$$H_1 = \frac{1}{2}(\boldsymbol{Y}_1^T \mathbf{Q}_1 \boldsymbol{Y}_1 + \boldsymbol{V}_1^T \mathbf{R}_1 \boldsymbol{V}_1) + \boldsymbol{L}_1^T(\mathbf{A}_1\boldsymbol{Y}_1 + \mathbf{B}_1\boldsymbol{V}_1 + \boldsymbol{D}_1^\circ) + \pi_1(y_1 - y_1^\circ) + \pi_3(y_3 - y_3^\circ) + \beta_1(v_1 - v_1^\circ) + \beta_2(v_2 - v_2^\circ) \quad (63)$$

where $\boldsymbol{L}_1^T = [\lambda_1, \lambda_3]$.

Second subsystem

$$\dot{\boldsymbol{Y}}_2 = \mathbf{A}_2\boldsymbol{Y}_2 + \mathbf{B}_2\boldsymbol{V}_2 + \boldsymbol{D}_2^\circ \quad (64)$$

The functional to be minimized is of the form

$$J_2 = \frac{1}{2}\int_0^{1.5} (\boldsymbol{Y}_2^T \mathbf{Q}_2 \boldsymbol{Y}_2 + \boldsymbol{V}_2^T \mathbf{R}_2 \boldsymbol{V}_2)\mathrm{d}t \quad (65)$$

where $\mathbf{Q}_2$ and $\mathbf{R}_2$ are unit matrices of the second order, and

$$\boldsymbol{Y}_2^T = [y_2, y_4] \quad \boldsymbol{V}_2^T = [v_3, v_4]$$

$$\mathbf{A}_2 = \begin{bmatrix} -6.7 & 0 \\ 0 & -1.7 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} -6 \times 10^{-4} & 0 \\ 0 & -2.5 \times 10^{-3} \end{bmatrix}$$

$$\boldsymbol{D}_2^\circ = \begin{bmatrix} f_2 \\ f_4 \end{bmatrix} - \mathbf{A}_2 \begin{bmatrix} y_2^\circ \\ y_4^\circ \end{bmatrix} - \mathbf{B}_2 \begin{bmatrix} v_3^\circ \\ v_4^\circ \end{bmatrix}$$

Hence

$$H_2 = \frac{1}{2}(\boldsymbol{Y}_2^T \mathbf{Q}_2 \boldsymbol{Y}_2 + \boldsymbol{V}_2^T \mathbf{R}_2 \boldsymbol{V}_2) + \boldsymbol{L}_2^T(\mathbf{A}_2\boldsymbol{Y}_2 + \mathbf{B}_2\boldsymbol{V}_2 + \boldsymbol{D}_2^\circ) + \pi_2(y_2 - y_2^\circ) + \pi_4(y_4 - y_4^\circ) + \beta_1(v_3 - v_3^\circ) + \beta_2(v_4 - v_4^\circ) \quad (66)$$

where $\boldsymbol{L}_2^T = [\lambda_2, \lambda_4]$.

The aim of the first level of the system under consideration is to independently determine y_i and v_i for both subsystems by solving the following Riccati equation

$$\dot{\mathbf{K}}_i = -\mathbf{Q}_i - \mathbf{K}_i\mathbf{A}_i - \mathbf{A}_i^{\mathrm{T}}\mathbf{K}_i + \mathbf{K}_i\mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{B}_i^{\mathrm{T}}\mathbf{K}_i \quad (67)$$

where $\mathbf{K}_i(1.5) = 0$ for $i = 1, 2$.

The matrix $\mathbf{K}_i$ could be calculated by substituting the values of the first subsystem. After some modifications one can obtain

$$\begin{aligned}
\dot{k}_{11} &= -1 + 2.1k_{11} + 0.0001k_{11}^2 + 0.0004k_{12}k_{21} \\
\dot{k}_{12} &= 1.05k_{12} + 0.0001k_{12}k_{11} + 0.0004k_{12}k_{22} \\
\dot{k}_{21} &= 1.05k_{21} + 0.0001k_{21}k_{11} + 0.0004k_{21}k_{22} \\
\dot{k}_{22} &= -1 + 0.0001k_{12}k_{21} + 0.0004k_{22}^2
\end{aligned} \quad (68)$$

where at the same time $k_{12} = k_{21}$.

Similarly for the second subsystem and for matrix $\mathbf{K}_2$ following relations hold

$$\begin{aligned}
\dot{k}_{11} &= -1 + 13.4k_{11} + 3.6 \times 10^{-7}k_{11}^2 \\
&\quad + 6.25 \times 10^{-6}k_{12}k_{21} \\
\dot{k}_{12} &= 8.4k_{12} + 3.6 \times 10^{-7}k_{11}k_{12} + 6.25 \times 10^{-6}k_{12}k_{22} \\
\dot{k}_{21} &= 8.4k_{21} + 3.6 \times 10^{-7}k_{11}k_{21} + 6.25 \times 10^{-6}k_{21}k_{22} \\
\dot{k}_{22} &= -1 + 3.4k_{22} + 3.6 \times 10^{-7}k_{12}k_{21} \\
&\quad + 6.25 \times 10^{-6}k_{22}^2
\end{aligned} \quad (69)$$

where also $k_{12} = k_{21}$. Differential equations (*68*) and (*69*) for calculations of the matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ were numerically solved using the Runge—Kutta method.

After elapsing of time $t = 8$ h the disturbances start to be present in the system. This could be expressed by help of the disturbances matrices according to the relation

$$\begin{aligned}
\frac{\mathrm{d}\boldsymbol{z}_i}{\mathrm{d}t} &= -\mathbf{A}_i^{\mathrm{T}}\boldsymbol{z}_i + \mathbf{K}_i\mathbf{B}_i\mathbf{R}_i^{-1}\mathbf{B}_i^{\mathrm{T}}\boldsymbol{z}_i + \mathbf{K}_i\mathbf{B}_i\mathbf{R}_i^{-1}\boldsymbol{\beta}_i \\
&\quad - \mathbf{K}_i\boldsymbol{D}_i - \boldsymbol{\pi}_i
\end{aligned} \quad (70)$$

For the first subsystem the following relations are valid

$$\begin{aligned}
\dot{z}_1 &= 1.05z_1 + 0.0001k_{11}z_1 + 0.0004k_{12}z_3 \\
&\quad + 0.02k_{11}\beta_2 - 0.01k_{11}\beta_1 + 1.05k_{11}x_1 \\
&\quad + 0.01k_{11}u_1 - 0.02k_{12}u_2 - 1.05k_{11}y_1^{\circ} \\
&\quad - 0.01k_{11}v_1^{\circ} + 0.02k_{12}v_2^{\circ} - \pi_1 \quad (71a) \\
\dot{z}_3 &= 0.0001k_{21}z_1 + 0.0004k_{22}z_3 \\
&\quad + 0.02k_{22}\beta_2 - 0.01k_{21}\beta_1 + 1.05k_{21}x_1 \\
&\quad + 0.01k_{22}u_2 - 0.02k_{22}u_2 - 1.05k_{21}y_1^{\circ} \\
&\quad - 0.01k_{21}v_1^{\circ} + 0.02k_{22}v_2^{\circ} - \pi_3 \quad (71b)
\end{aligned}$$

For the second subsystem following equations could be obtained

$$\begin{aligned}
\dot{z}_2 &= 6.7z_2 + 3.6 \times 10^{-7}k_{11}z_2 + 6.25 \times 10^{-6}k_{12}z_4 \\
&\quad - 6 \times 10^{-4}k_{11}\beta_1 - 2.5 \times 10^{-3}k_{12}\beta_2 + 6.7k_{11}x_2 \\
&\quad + 1.7k_{12}x_4 + 6 \times 10^{-4}k_{11}u_1 \\
&\quad + 2.5 \times 10^{-3}k_{12}u_2 - 6.7k_{11}y_2^{\circ} - 1.7k_{12}y_4^{\circ} \\
&\quad - 6 \times 10^{-4}k_{11}v_3^{\circ} - 2.5 \times 10^{-3}k_{12}v_4^{\circ} - \pi_2 \quad (72a) \\
\dot{z}_4 &= 1.7z_4 + 3.6 \times 10^{-7}k_{21}z_2 + 6.25 \times 10^{-6}k_{22}z_4 \\
&\quad - 6 \times 10^{-4}k_{21}\beta_1 - 2.5 \times 10^{-3}k_{22}\beta_2 + 6.7k_{11}x_2 \\
&\quad + 1.7k_{12}x_4 + 6 \times 10^{-4}k_{11}u_1 \\
&\quad + 2.5 \times 10^{-3}k_{12}u_2 - 6.7k_{11}y_2^{\circ} - 1.7k_{12}y_4^{\circ} \\
&\quad - 6 \times 10^{-4}k_{11}v_3^{\circ} - 2.5 \times 10^{-3}k_{12}v_4^{\circ} - \pi_4 \quad (72b)
\end{aligned}$$

with $z_i(1.5) = 0$ for $i = 1$ to 4.

In simulation of the whole system the original initial conditions for the disturbances must be found again by the use of an integration in the backward time.

The quantities $\boldsymbol{y}_i^{\circ}, \boldsymbol{v}_i^{\circ}, \boldsymbol{\pi}_i$, and $\boldsymbol{\beta}_i$ are given by the second level according to the following relations.

$$\boldsymbol{v}_i = -\mathbf{R}_i^{-1}(\mathbf{B}_i^{\mathrm{T}}(\mathbf{K}_i\boldsymbol{y}_i + \boldsymbol{z}_i) + \boldsymbol{\beta}_i) \quad (73)$$

$$\dot{\boldsymbol{y}}_i = \mathbf{A}_i\boldsymbol{y}_i + \mathbf{B}_i\boldsymbol{v}_i + \boldsymbol{D}_i \quad (74)$$

For the first subsystem it holds

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = -\begin{bmatrix} -0.01 & 0 \\ 0 & 0.02 \end{bmatrix} \cdot \left(\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_3 \end{bmatrix} \right) + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \quad (75)$$

By modification of this expression the quantities v_1 and v_2 could be obtained

$$v_1 = 0.01(k_{11}y_1 + k_{12}y_3 + z_1) - \beta_1$$

$$v_2 = -0.02(k_{21}y_1 + k_{22}y_3 + z_3) - \beta_2$$

The quantities y_1 and y_3 could be evaluated by a modification of the following equation.

$$\begin{aligned}
\begin{bmatrix} \dot{y}_1 \\ \dot{y}_3 \end{bmatrix} &= \begin{bmatrix} -1.05 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_3 \end{bmatrix} \\
&\quad + \begin{bmatrix} -0.01 & 0 \\ 0 & 0.02 \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\
&\quad + \begin{bmatrix} -1.05 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} \\
&\quad + \begin{bmatrix} -0.01 & 0 \\ 0 & 0.02 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\
&\quad - \begin{bmatrix} -1.05 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1^{\circ} \\ y_3^{\circ} \end{bmatrix} \\
&\quad - \begin{bmatrix} -0.01 & 0 \\ 0 & 0.02 \end{bmatrix} \cdot \begin{bmatrix} v_1^{\circ} \\ v_2^{\circ} \end{bmatrix}
\end{aligned} \quad (76)$$

$$\begin{aligned}
\dot{y}_1 &= -1.05y_1 - 0.01v_1 - 1.05x_1 - 0.01u_1 \\
&\quad + 1.05y_1^{\circ} + 0.01v_1^{\circ} \\
\dot{y}_3 &= 0.02v_2 + 0.02u_2 - 0.02v_2^{\circ}
\end{aligned}$$

For the second subsystem

$$v_3 = 6 \times 10^{-4}(k_{11}y_2 + k_{12}y_4 + z_2) - \beta_1$$
$$v_4 = 2.5 \times 10^{-3}(k_{21}y_2 + k_{22}y_4 + z_4) - \beta_2$$
$$\dot{y}_2 = -6.7(y_2 + x_2 - y_2^\circ) - 6 \times 10^{-4}(v_3 + u_1 - v_3^\circ)$$
$$\dot{y}_4 = -1.7(y_4 + x_4 - y_4^\circ) - 2.5 \times 10^{-3}(v_4 + u_2 - v_4^\circ)$$

The second level sets quantities $\boldsymbol{y}_i^\circ, \boldsymbol{v}_i^\circ, \boldsymbol{\pi}_i$, and $\boldsymbol{\beta}_i$ on the basis of optimality conditions

$$\frac{\partial H}{\partial \boldsymbol{\pi}} = \mathbf{0} = \boldsymbol{y}^\circ - \boldsymbol{y}$$
$$\frac{\partial H}{\partial \boldsymbol{\beta}} = \mathbf{0} = \boldsymbol{v}^\circ - \boldsymbol{v} \qquad (77)$$

from which it follows that $\boldsymbol{y}^\circ = \boldsymbol{y}$ and $\boldsymbol{v}^\circ = \boldsymbol{v}$.

$$\frac{\partial H}{\partial \boldsymbol{y}^\circ} = \mathbf{0} = \boldsymbol{\pi} - [\mathbf{A} - \mathbf{A}^{\mathrm{T}}] \cdot \boldsymbol{L}(t) \qquad (78)$$
$$\frac{\partial H}{\partial \boldsymbol{v}^\circ} = \mathbf{0} = \boldsymbol{\beta} - [\mathbf{B} - \mathbf{B}^{\mathrm{T}}] \cdot \boldsymbol{L}(t) \qquad (79)$$

where $\mathbf{A}$ is the total matrix of the full system, and $\boldsymbol{L}$ is the vector of Lagrangian multipliers.

$$\mathbf{A} = \begin{bmatrix} -1.05 & 0 & 0 & 0 \\ 0.003 & -6.7 & 0 & 1.7 \\ 0 & 7.4 & 0 & 0 \\ -0.003 & 0 & 0 & -1.7 \end{bmatrix} \qquad \boldsymbol{L} = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}$$

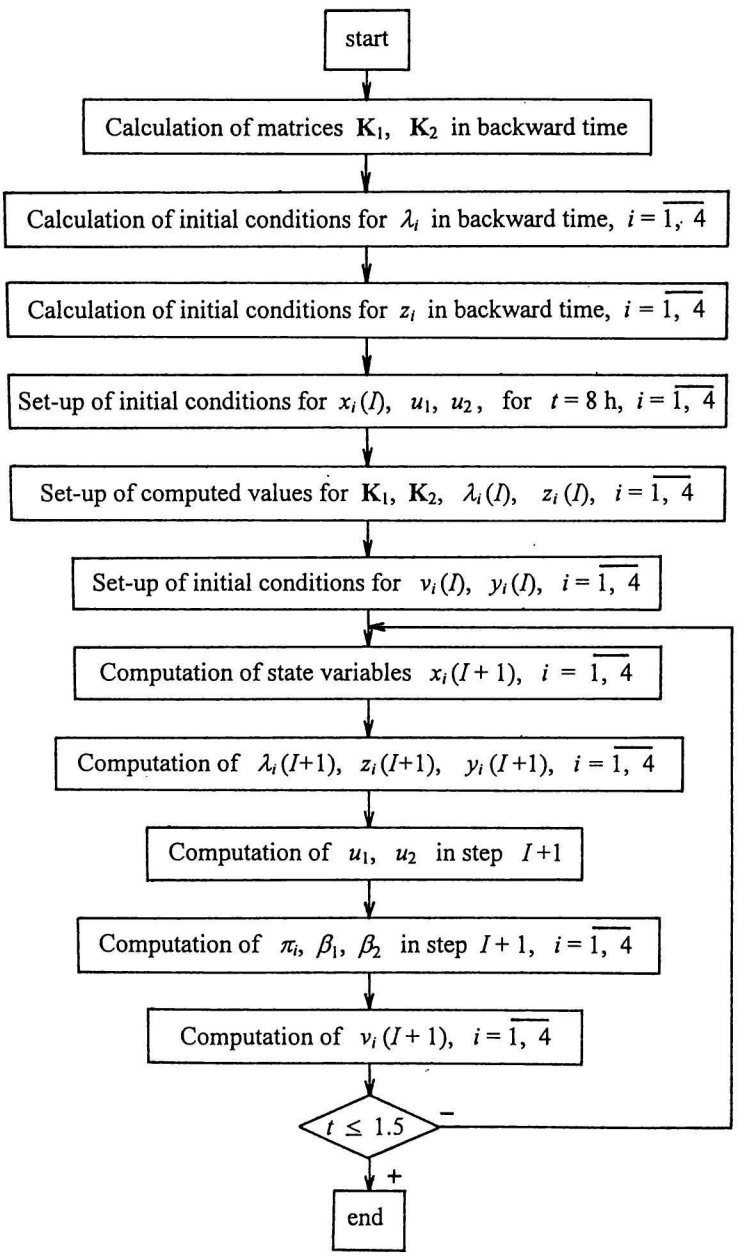

Fig. 3. Algorithm of the calculation according to predictive coordination.

After substitution and rearranging following relations could be obtained

$$\begin{aligned}\pi_1 &= -0.003\lambda_2 + 0.003\lambda_4\\ \pi_2 &= 0.003\lambda_1 - 7.4\lambda_3 - 1.7\lambda_4\\ \pi_3 &= 7.4\lambda_2\\ \pi_4 &= -0.003\lambda_1 - 1.7\lambda_2\end{aligned} \tag{80}$$

From optimality conditions it further follows

$$\mathbf{B} = \begin{bmatrix} 0.01 & 0\\ -6\times10^{-4} & 0\\ 0 & 0.02\\ 0 & -2.5\times10^{-3}\end{bmatrix} \quad \boldsymbol{L} = \begin{bmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\end{bmatrix} \tag{81}$$

After some rearranging

$$\begin{aligned}\beta_1 &= 0.01\lambda_1 + 6\times10^{-4}\lambda_2 + 0.02\lambda_3\\ &\quad - 2.5\times10^{-3}\lambda_4\\ \beta_2 &= -0.01\lambda_1 - 6\times10^{-4}\lambda_2 - 0.02\lambda_3\\ &\quad + 2.5\times10^{-3}\lambda_4\end{aligned} \tag{82}$$

The prediction error has the following form

$$\begin{aligned}e = \Bigg(\int_0^{1.5} \Bigg\{ \sum_{i=1}^{4} \Big[&\left(y_i^{o(I+1)} - y_i^{o(I)}\right)^2\\ &+ \left(v_i^{o(I+1)} - v_i^{o(I)}\right)^2\\ &+ \left(\pi_i^{(I+1)} - \pi_i^{(I)}\right)^2 \Big]\\ &+ \sum_{i=1}^{2} \left(\beta_i^{(I+1)} - \beta_i^{(I)}\right)^2 \Bigg\} \mathrm{d}t \Bigg)^{\frac{1}{2}} < 10^{-6}\end{aligned} \tag{83}$$

Simulation was carried out according to the algorithm depicted in Figs. 2 and 3.

The results are graphically presented in Figs. 4 to 9, where the time dependences of state and command variables are depicted. The hierarchical control of biotechnological complex is illustrated in Fig. 10.

More detailed information on the control strategy for industry bioprocesses can be found in [6]. On direct adaptive control of fermentation processes the reader is referred to [7].

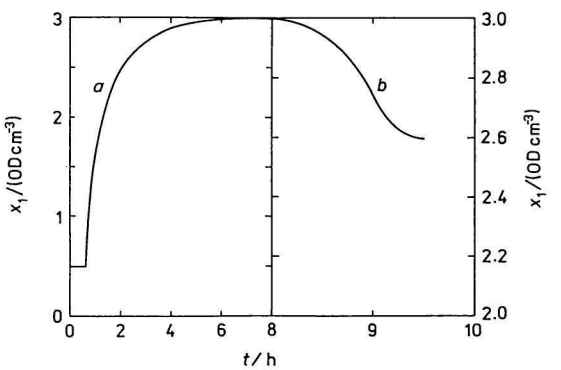

Fig. 4. Optimal course of the state variable x_1. *a*) Objective coordination; *b*) predictive coordination.

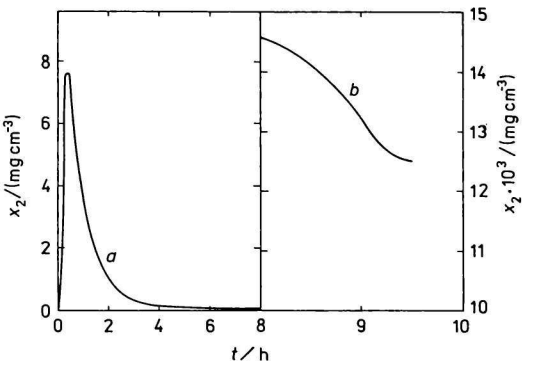

Fig. 5. Optimal course of the state variable x_2. *a*) Objective coordination; *b*) predictive coordination.

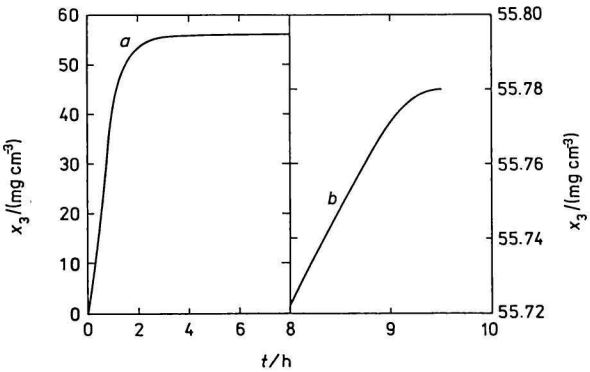

Fig. 6. Optimal course of the state variable x_3. *a*) Objective coordination; *b*) predictive coordination.

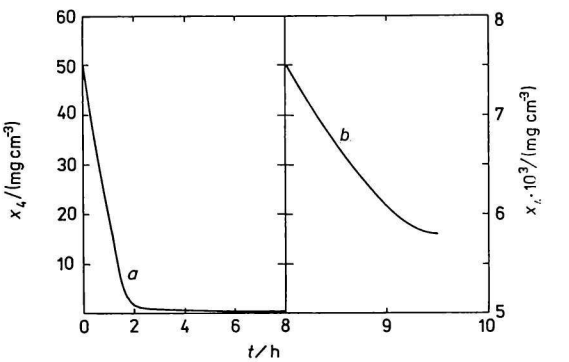

Fig. 7. Optimal course of the state variable x_4. *a*) Objective coordination; *b*) predictive coordination.

Program Realization

For the solution of the differential equations the standard Runge—Kutta procedure was used.

In the case of using the prediction coordination method the coefficients of the matrix $\mathbf{K}_i$ were calculated first. Also in this case the standard Runge—Kutta procedure was used in the program.

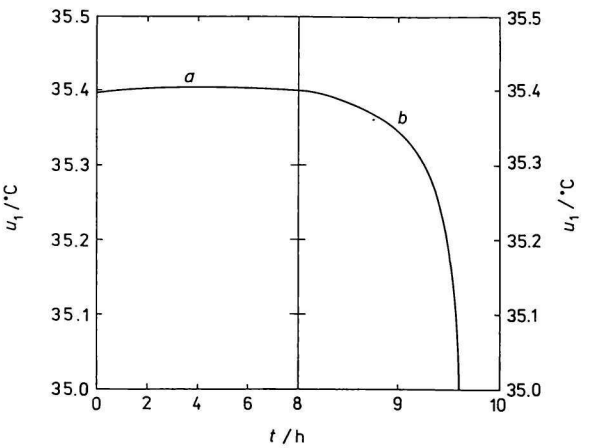

Fig. 8. Optimal course of the command variable u_1. *a*) Objective coordination; *b*) predictive coordination.

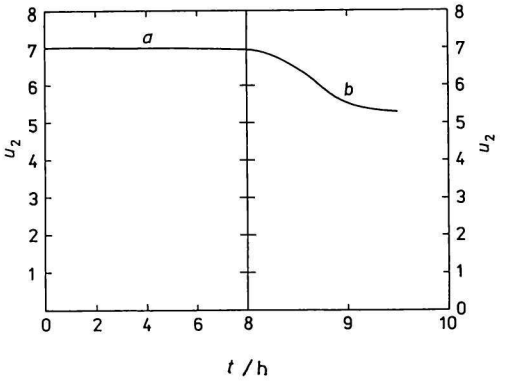

Fig. 9. Optimal course of the command variable u_2. *a*) Objective coordination; *b*) predictive coordination.

RESULTS AND DISCUSSION

Three decomposition possibilities were examined and only one, for which the following results are introduced, gives realizable results. Fig. 11 shows optimal trajectories for state and command variables valid for the first and second scheme.

From Fig. 11 it is also evident that the hierarchical approach brings higher D-gluconic acid concentration in comparison to one-level technique. As can be seen the optimal profiles of control variables for both subsystems are approximately the same. There are shown only optimum profiles for the first subsystem.

The reason for choosing the second alternative is obtaining optimum solution.

Nevertheless by suitable selection of weight matrix **Q** it is possible to achieve speeding up the convergence of state variables to the desired values. We came to the conclusion that the hierarchical approach using microprocessor may be regarded for an effective technique for obtaining optimum solution for some fermentation processes.

It was observed that the hierarchical approach (multilevel technique) is more advantageous and more attractive than the one-level approach as

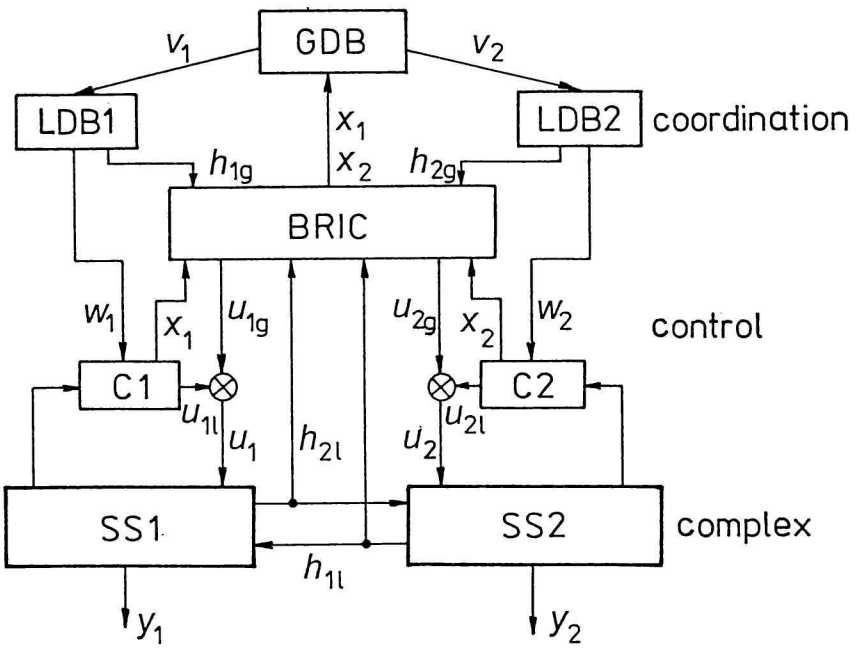

Fig. 10. Hierarchical control of biotechnological complex. SS1, SS2 – subsystems; C1, C2 – controllers; BRIC – block for regarding of interaction couplings; LDB1, LDB2 – local decision-making block; GDB – global decision-making block; h_{1g}, h_{2g} – global coupling variables; h_{1l}, h_{2l} – local coupling variables; u_{1g}, u_{2g} – interaction quantities; u_{1l}, u_{2l} – local command variables; v_1, v_2 – coordination quantities; w_1, w_2 – desired values; x_1, x_2 – state variables; y_1, y_2 – output variables.

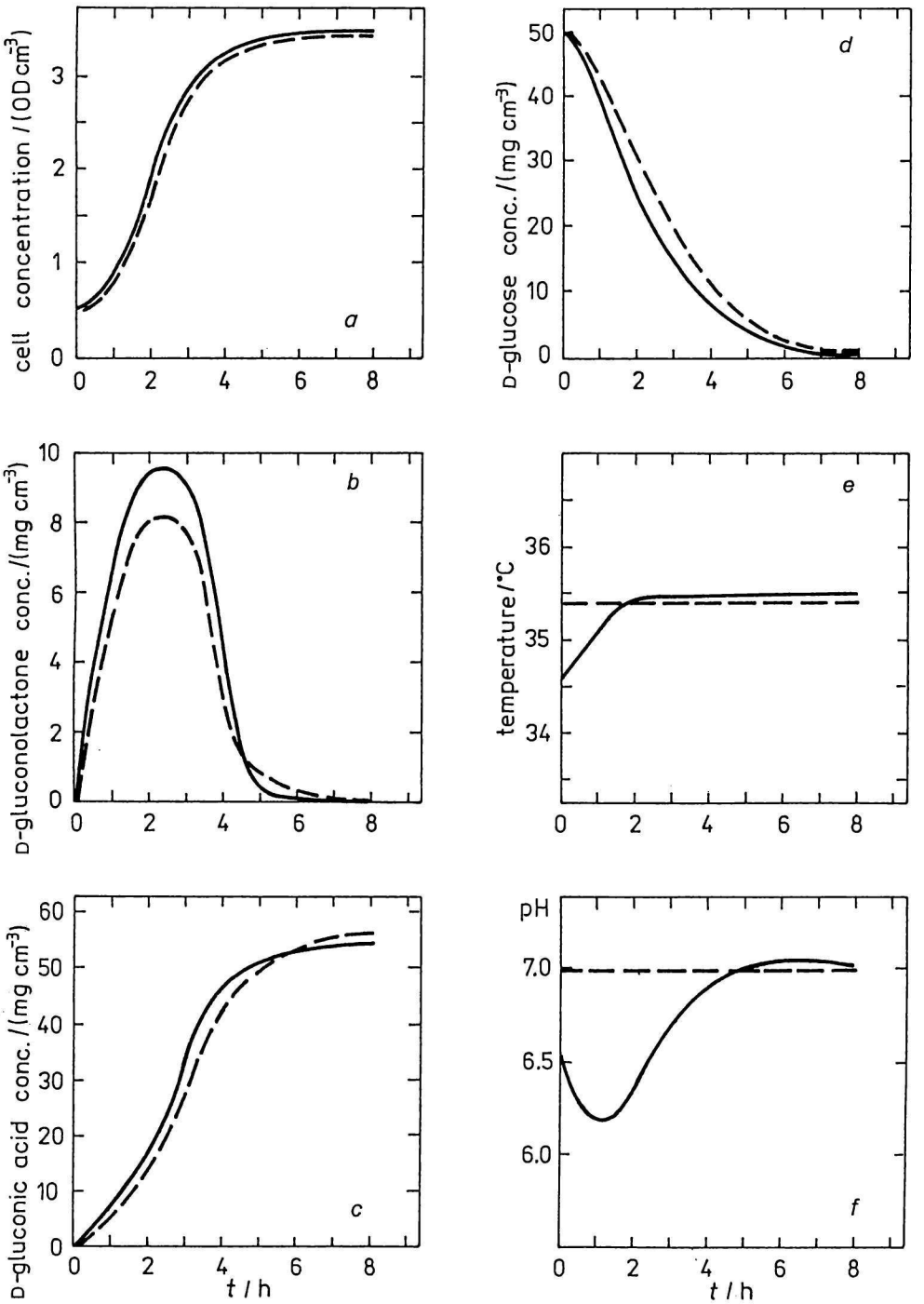

Fig. 11. Courses of state variables x_1, x_2, x_3, x_4 and of command variables u_1 and u_2. — 1st scheme, - - - 2nd scheme.

a) at any level lower-degree subproblems are manipulated,

b) the computations are more accurate while the manipulation with lower-degree problems secures that the rounding errors are less,

c) programs demand roughly one half of the memory capacity in comparison to the one-level approach,

d) higher yield of D-gluconic acid is achieved (the second scheme).

The results demonstrate that the hierarchical approach may be regarded as an effective technique for obtaining the optimal solution for fermentation process. This control is more advantageous than the one-level method. As can be seen from presented graphs, it provides a higher yield of D-gluconic acid.

The decomposition coordination methods were successfully applied for optimization and control of fermentation processes, which can be considered as

complex systems.

The model described is restricted only to four state variables, but could be expanded to include more variables, *e.g.* the oxygen concentration or stirrer revolutions. Introduced knowledge will enable application of higher forms of control to the control of complex systems (complexes).

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