

An Approach to Adaptive Control of a CSTR

^aP. DOSTÁL, ^bM. BAKOŠOVÁ, and ^aV. BOBÁL

^aDepartment of Control Theory, Institute of Information Technologies,
Tomas Bata University in Zlín, CZ-762 72 Zlín
e-mail: dostalp@ft.utb.cz, bobal@ft.utb.cz

^bDepartment of Information Engineering and Process Control, Faculty of Chemical and Food Technology,
Slovak University of Technology, SK-812 37 Bratislava
e-mail: bakosova@chtf.stuba.sk

Received 17 October 2002

The paper deals with continuous-time adaptive control of a continuous stirred tank reactor. A nonlinear model of the plant is approximated by an external linear model with recursively estimated parameters. Both, one degree of freedom and two degrees of freedom control system configurations are considered. Resulting controllers ensure internal properness and stability of the control system as well as asymptotic tracking of the step reference and step load disturbance attenuation. The design procedure is based on the polynomial method. An application of the LQ control technique yields a control law penalizing the control input changes. The adaptive control is tested by simulation means.

Continuous stirred tank reactors (CSTRs) are central components of many plants in the chemical industry. From the system-engineering point of view, they belong to a class of nonlinear systems where both steady-state and dynamic behaviour are nonlinear. This often causes conventional control methods to be unsatisfactory. Moreover, when strongly exothermic reactions take place in the reactor, the reactant temperature may exhibit high sensitivity to control input changes. Therefore, the control system should be able to accept both the system nonlinearity as well as the sensitivity. This requirement can be met by using adaptive control with parallel restriction of control input changes.

The paper presents one approach to continuous-time adaptive control of a CSTR. A nonlinear model of the process is approximated by an external linear model (ELM) with recursively estimated parameters. Since the controlled process input and output derivatives cannot be measured directly, differential filters and filtered variables are established substituting the primary variables, see *e.g.* [1]. The filtered variables, inclusive of their derivatives are then measured at discrete time intervals and used in the recursive identification procedure.

The one degree of freedom (1DOF) and two degrees of freedom (2DOF) control system configurations are considered, see *e.g.* [2]. The reference signal is considered to be from a class of step or exponential functions.

For the controller design, the polynomial approach is used, see *e.g.* [3, 4]. Requirements on the control system properties are formulated as its internal proper-

ness and stability and asymptotic tracking of a reference and step load disturbance attenuation. Using the polynomial approach, the resulting controllers are obtained by a solution of polynomial Diophantine equations with a stable polynomial on the right sides. Further, the LQ control technique is applied to restrict the control input changes. This procedure is described in [5–7]. Stable right sides of polynomial equations are then given by spectral factorizations. This procedure yields strictly proper controllers.

The control of a CSTR based on the above procedure is tested on the nonlinear model described by four nonlinear differential equations, see *e.g.* [8, 9].

THEORETICAL

ELM is chosen on the basis of any preliminary knowledge of dynamic behaviour of the controlled system. This model is described by differential equation in the time domain

$$a(\sigma)y(t) = b(\sigma)u(t) \quad (1)$$

where σ is the derivative operator and a , b are polynomials in σ . Considering zero initial conditions and using the Laplace transform, the ELM is represented in the complex domain by

$$Y(s) = \frac{b(s)}{a(s)}U(s) \quad (2)$$

where s is the complex variable and Y and U denote the Laplace transforms of y and u . Both a and b are now coprime polynomials in the form

$$a(s) = \sum_{i=0}^{\deg a} a_i s^i, \quad b(s) = \sum_{j=0}^{\deg b} b_j s^j \quad (3)$$

where $a(s)$ is a monic polynomial ($a_{\deg a} = 1$). Assuming the condition $\deg b \leq \deg a$, the transfer function in eqn (2)

$$G(s) = \frac{b(s)}{a(s)} \quad (4)$$

is proper.

The method of ELM parameter estimation has been analyzed *e.g.* in [1]. Since the derivatives of both input and output variables cannot be directly measured, filtered variables u_f and y_f are established as the outputs of filters

$$c(\sigma) u_f(t) = u(t), \quad c(\sigma) y_f(t) = y(t), \quad (5)$$

where $c(\sigma)$ is a stable polynomial in σ which fulfils the condition $\deg c \geq \deg a$. Note that time constants of the filters $c(\sigma)$ must be smaller than time constants of the model. Since the latter are unknown at the beginning of the estimation procedure, it is necessary to choose the filter time constants, selected a priori, sufficiently small.

It can be easily proven that the transfer behaviour between the filtered and between nonfiltered variables is equivalent. This fact enables to employ the filtered variables for the ELM parameter estimation. If these are computed *via* filters (5) in discrete time intervals $t_k = k \tau_s, k = 0, 1, 2, \dots$, where τ_s is the sampling period, then, denominating $\deg a = n$ and $\deg b = m$, and establishing the regression vector

$$\Phi^T(t_k) = [-y_f(t_k), -y_f^{(1)}(t_k), \dots, -y_f^{(n-1)}(t_k), u_f(t_k), u_f^{(1)}(t_k), \dots, u_f^{(m)}(t_k)] \quad (6)$$

the vector of ELM parameters

$$\Theta^T(t_k) = [a_0, a_1, \dots, a_{n-1}, b_0, b_1, \dots, b_m] \quad (7)$$

can be recursively estimated from the equation

$$y_f^{(n)}(t_k) = \Theta^T(t_k) \Phi(t_k) + \varepsilon(t_k) \quad (8)$$

where ε expresses the difference between initial conditions of filtered and nonfiltered variables.

Considered control system configurations are depicted in Figs. 1 and 2. The 1DOF control configuration contains only a feedback controller. The controller in the 2DOF control configuration contains also a feedforward part. Here, G is the ELM with the transfer function (4), Q and R are the feedback and feedforward con-

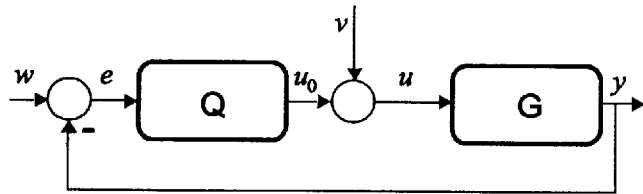


Fig. 1. 1DOF control system configuration.

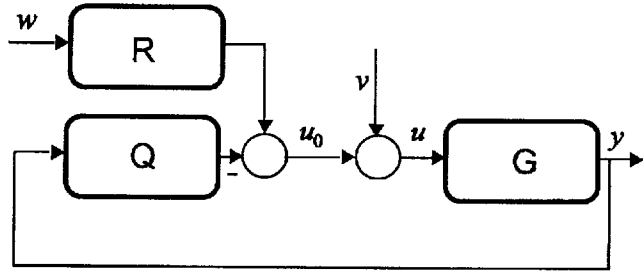


Fig. 2. 2DOF control system configuration.

trollers, respectively. In both schemes, w is the reference signal, v is the load disturbance, and u is the control variable. The reference w is considered to be a step or exponential function with a transform function expressed in the form

$$W(s) = \frac{h_w(s)}{s f_w(s)} \quad (9)$$

where h_w, f_w are polynomials and f_w is a stable polynomial. The load disturbance v is assumed as a step function. For such reference and disturbance, transfer functions of the controllers Q and R have the form

$$Q(s) = \frac{q(s)}{s p(s)}, \quad R(s) = \frac{r(s)}{s p(s)} \quad (10)$$

where q, r , and p are polynomials in s .

The controller design described in this section is based on the polynomial approach. General conditions required to govern the control system properties are formulated as internal properness and stability of the control system, asymptotic tracking of the reference and load disturbance attenuation. The procedure to derive admissible controllers can be carried out as follows.

A feedback controller given by a solution of the polynomial equation

$$a(s)sp(s) + b(s)q(s) = d(s) \quad (11)$$

with a stable polynomial d on the right side ensures the stability and load disturbance attenuation in both 1DOF and 2DOF control system configurations, and, in addi-

Table 1. Parameters, Input and Initial Conditions

$V_r = 1.2 \text{ m}^3$	$A = 5.5 \text{ m}^2$	$c_{Af} = 2.85 \text{ kmol m}^{-3}$
$V_c = 0.64 \text{ m}^3$	$U = 43.5 \text{ kJ m}^{-2} \text{ min}^{-1} \text{ K}^{-1}$	$c_{Bf} = 0 \text{ kmol m}^{-3}$
$Q_r = 0.08 \text{ m}^3 \text{ min}^{-1}$	$k_{10} = 5.616 \times 10^{16} \text{ min}^{-1}$	$T_{rf} = 323 \text{ K}$
$Q_c^s = 0.03 \text{ m}^3 \text{ min}^{-1}$	$k_{20} = 1.128 \times 10^{18} \text{ min}^{-1}$	$T_{cf} = 293 \text{ K}$
$\rho_r = 985 \text{ kg m}^{-3}$	$E_1/R = 13477 \text{ K}$	$c_A^s = 0.1649 \text{ kmol m}^{-3}$
$\rho_c = 998 \text{ kg m}^{-3}$	$E_2/R = 15290 \text{ K}$	$c_B^s = 0.9435 \text{ kmol m}^{-3}$
$c_{pr} = 4.05 \text{ kJ kg}^{-1} \text{ K}^{-1}$	$h_1 = 4.8 \times 10^4 \text{ kJ kmol}^{-1}$	$T_r^s = 350.19 \text{ K}$
$c_{pc} = 4.18 \text{ kJ kg}^{-1} \text{ K}^{-1}$	$h_2 = 2.2 \times 10^4 \text{ kJ kmol}^{-1}$	$T_c^s = 330.55 \text{ K}$

tion, asymptotic tracking in the 1DOF control configuration. The asymptotic tracking in the 2DOF control system is provided by the feedforward controller given by a solution of the polynomial equation

$$z(s)s + b(s)r(s) = d(s) \quad (12)$$

where $z(s)$ is an unknown additional polynomial.

The control system satisfies the condition of internal properness if the transfer functions of all components are proper. Degrees of the controller polynomials have then to fulfil inequalities

$$\deg q \leq \deg p + 1, \deg r \leq \deg p + 1 \quad (13)$$

Taking into account relations (13), the condition $\deg b \leq \deg a$, and the solvability of eqns (11) and (12), it is possible to derive that the degrees of polynomials q and r are

$$\deg q = \deg a, \deg p \geq \deg a - 1, \deg r = 0 \quad (14)$$

Moreover, the equality $r_0 = q_0$ can be easily proved.

The controller parameters are then obtained via solutions of polynomial equations (11), (12) and depend upon coefficients of the polynomial d . The next problem is finding a stable polynomial d providing the acceptable stabilizing controllers.

For application of LQ control technique, the polynomial d is sought as a product of two stable polynomials g and n in the form

$$d(s) = g(s)n(s) \quad (15)$$

The first polynomial g is calculated from spectral factorization

$$(sa(s))^* \varphi(sa(s)) + b^*(s)\mu b(s) = g^*(s)g(s) \quad (16)$$

where the asterisk denotes a conjugate polynomial. The polynomial g is well known from the LQ control theory (see e.g. [2, 5]). There, the poles which minimize an integrand in the quadratic cost function

$$J = \frac{1}{2\pi j} \int_{-j\infty}^{j\infty} \left\{ E^*(s)\mu E(s) + \tilde{U}^*(s)\varphi\tilde{U}(s) \right\} ds \quad (17)$$

are roots of the polynomial g . Here, $E(s)$ is the transform of the tracking error $e(t)$, $\tilde{U}(s)$ denotes the transform of the control input derivative and μ and φ are weighting coefficients.

The polynomial n ensures properness of the controller and it is given as a stable part of spectral factorization

$$n^*(s)n(s) = a^*(s)a(s) \quad (18)$$

Polynomials q , r , and p are then given by solutions of polynomial equations

$$a(s)s p(s) + b(s)q(s) = g(s)n(s) \quad (19)$$

$$\frac{z(s)s + b(s)r(s) = g(s)n(s)}{1} \quad (20)$$

The transfer functions (10) are strictly proper. The degrees of the right sides of eqns (19) and (20) are given by $\deg[g(s)n(s)] = 2\deg a(s) + 1$. Taking into account condition (14) and the relation $\deg[sp(s)] = \deg[g(s)n(s)] - \deg a(s) = \deg a(s) + 1$, the strict properness of both transfer functions in eqn (10) is evident.

SIMULATIONS

Consider a CSTR with the first-order consecutive exothermic reactions according to the scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$ and with a perfectly mixed cooling jacket. Using usual simplifications, the model of the CSTR is described by four nonlinear differential equations

$$\frac{dc_A}{dt} = -\left(\frac{Q_r}{V_r} + k_1\right)c_A + \frac{Q_r}{V_r}c_{Af} \quad (21)$$

$$\frac{dc_B}{dt} = -\left(\frac{Q_r}{V_r} + k_2\right)c_B + k_1c_A + \frac{Q_r}{V_r}c_{Bf} \quad (22)$$

$$\frac{dT_r}{dt} = \frac{h_r}{(\rho c_p)_r} + \frac{Q_r}{V_r}(T_{rf} - T_r) + \frac{A_h U}{V_r(\rho c_p)_r}(T_c - T_r) \quad (23)$$

$$\frac{dT_c}{dt} = \frac{Q_c}{V_c}(T_{cf} - T_c) + \frac{A_h U}{V_c(\rho c_p)_c}(T_r - T_c) \quad (24)$$

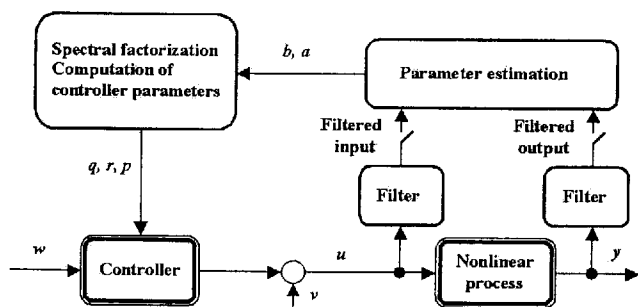


Fig. 3. Adaptive control scheme.

with initial conditions $c_A(0) = c_A^s$, $c_B(0) = c_B^s$, $T_r(0) = T_r^s$, and $T_c(0) = T_c^s$. Here, t is time, c are concentrations, T are temperatures, V are volumes, ρ are densities, c_p are specific heat capacities, Q are volumetric flow rates, A_h is the heat exchange surface area, and U is the heat transfer coefficient. The subscripts r denote the reactant mixture, c the coolant, f feed (inlet) values and the superscript s the steady-state values.

The reaction rates and the reaction heat are expressed as

$$k_j = k_{0j} \exp\left(\frac{-E_j}{RT_r}\right), j = 1, 2 \quad (25)$$

$$h_r = h_1 k_1 c_A + h_2 k_2 c_B \quad (26)$$

where k_0 are pre-exponential factors, E are activation energies, and h are reaction enthalpies. The values of all parameters, feed values, and steady-state values are given in Table 1.

The aim is to control temperature of reaction mixture by flow rate of coolant. For purposes of control synthesis, the controlled variable and the control variable are defined as

$$y(t) = T_r(t) - T_r^s, u(t) = 10 \frac{Q_c(t) - Q_c^s}{Q_c^s} \quad (27)$$

These expressions enable to obtain variables of approximately the same magnitude.

The adaptive control system is shown in Fig. 3. According to the scheme, the adaptive control can be realized in the following steps.

1. The ELM (1) of a controlled nonlinear process is chosen using a previous analysis of the process dynamic behaviour.
2. Both control input and controlled output (27) enter into filters (5).
3. The filtered variables inclusive of their derivatives are measured at filter outputs, sampled and subsequently used for the ELM parameter estimation according to eqn (8).
4. Coefficients of the stable polynomials g and n are computed using formulas derived from spectral factorizations (16) and (18).

5. The controller parameters are computed from a set of equations originating from solutions of eqns (19) and (20) and periodically readjusted.

RESULTS AND DISCUSSION

For the adaptive control simulation, the following changes of reference values were tracked: $w(t) = 4 \cdot (1 - \exp(-0.1 \cdot t))$ K for $0 \text{ min} \leq t < 150 \text{ min}$, $w(t) = -2$ K for $150 \text{ min} \leq t < 270 \text{ min}$ and $w(t) = 4$ K for $t \geq 270 \text{ min}$. The boundary of the control input was used within the range $-6.5 \leq u(t) \leq 3.5$. Using some results of the CSTR previous dynamic analysis, the second-order ELM expressed by the transfer function

$$G(s) = \frac{b(s)}{a(s)} = \frac{b_0}{s^2 + a_1 s + a_0} \quad (28)$$

has been chosen. The unit weighting coefficient μ and various φ coefficients were used.

The control simulation consisted of the following steps.

1. The filtered variables and their derivatives were obtained from filters

$$\dot{y}_f(t) + c_1 \dot{y}_f(t) + c_0 y_f(t) = y(t) \quad (29)$$

$$\ddot{u}_f(t) + c_1 \dot{u}_f(t) + c_0 u_f(t) = u(t) \quad (30)$$

2. The ELM parameters [a_0 , a_1 , b_0] were recursively estimated from the equation

$$\dot{y}_f(t_k) = b_0 u_f(t_k) - a_0 y_f(t_k) - a_1 \dot{y}_f(t_k) + \varepsilon(t_k) \quad (31)$$

in discrete time intervals $t_k = k \tau_s$ with the sampling period $\tau_s = 0.5 \text{ min}$. Here, the identification procedure based on the least-squares method [10] was used.

3. The polynomials g and n in spectral factorizations (16) and (18) have the form

$$g(s) = g_3 s^3 + g_2 s^2 + g_1 s + g_0 \quad (32)$$

$$g(s) = s^2 + n_1 s + n_0 \quad (33)$$

and their coefficients were computed as

$$g_0 = \sqrt{\mu b_0^2}, \quad g_1 = \sqrt{2g_2 g_0 + \varphi a_0^2}, \quad (34)$$

$$g_2 = \sqrt{2g_3 g_1 + \varphi(a_1^2 - 2a_0)}, \quad g_3 = \sqrt{\varphi}$$

$$n_0 = \sqrt{a_0^2}, \quad n_1 = \sqrt{2n_0 + a_1^2 - 2a_0} \quad (35)$$

4. The polynomial $d = gn$ on the right sides of polynomial equations (19) and (20) has the form

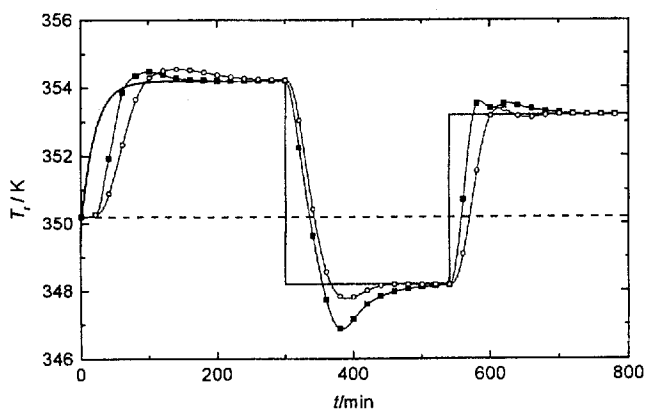


Fig. 4. 1DOF configuration - controlled temperature responses to the reference value changes (—) around steady state (---) for $\phi = 0.25$ (■) and $\phi = 25$ (○).

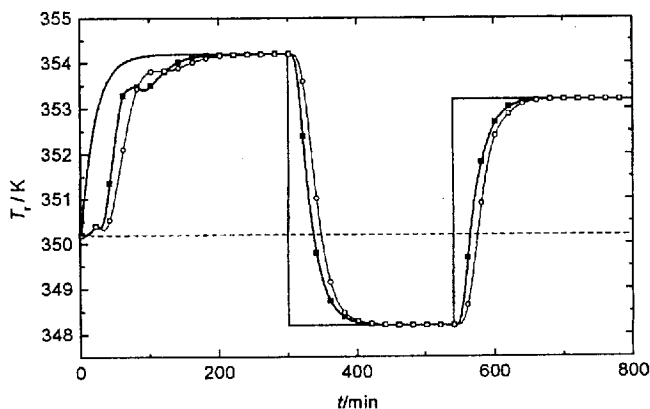


Fig. 5. 2DOF configuration - controlled temperature responses to the reference value changes (—) around steady state (---) for $\phi = 0.25$ (■) and $\phi = 25$ (○).

$$d(s) = d_5 s^5 + d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 \quad (36)$$

with coefficients

$$d_5 = g_3, \quad d_4 = g_3 n_1 + g_2 \quad (37)$$

$$d_3 = g_3 n_0 + g_2 n_1 + g_1$$

$$d_2 = g_2 n_0 + g_1 n_1 + g_0, \quad d_1 = g_1 n_0 + g_0 n_1 \quad (38)$$

$$d_0 = g_0 n_0$$

5. The degrees of polynomials q , r , and p were determined in accordance with relations (14) as

$$\deg q = 2, \quad \deg r = 0, \quad \deg p = 2 \quad (39)$$

and the transfer functions of the resulting controllers then take the form

$$Q(s) = \frac{q_2 s^2 + q_1 s + q_0}{s(p_2 s^2 + p_1 s + p_0)}$$

$$R(s) = \frac{r_0}{s(p_2 s^2 + p_1 s + p_0)} \quad (40)$$

The controller parameters were computed as

$$p_2 = d_5, p_1 = d_4 - a_1 p_2, p_0 = d_3 - a_1 p_1 - a_0 p_2 \quad (41)$$

$$q_2 = \frac{1}{b_0} (d_2 - a_1 p_0 - a_0 p_1), \quad q_1 = \frac{1}{b_0} (d_1 - a_0 p_0), \quad (42)$$

$$q_0 = \frac{1}{b_0} d_0, \quad r_0 = q_0$$

The control law for the 1DOF configuration is given in the time domain as

$$p_2 \ddot{u} + p_1 \dot{u} + p_0 u = q_2 \ddot{e} + q_1 \dot{e} + q_0 e \quad (43)$$

and for the 2DOF configuration as

$$p_2 \ddot{u} + p_1 \dot{u} + p_0 u = r_0 w - q_2 \ddot{y} - q_1 \dot{y} - q_0 y \quad (44)$$

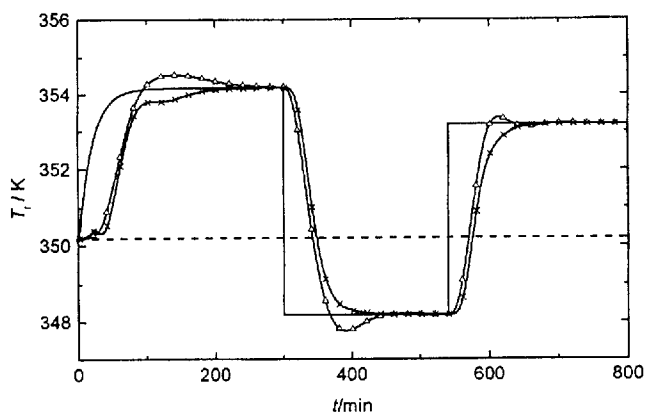


Fig. 6. Comparison of controlled temperature responses in 1DOF (Δ) and 2DOF (X) configurations to the reference value changes (—) around steady state (---) for $\phi = 25$.

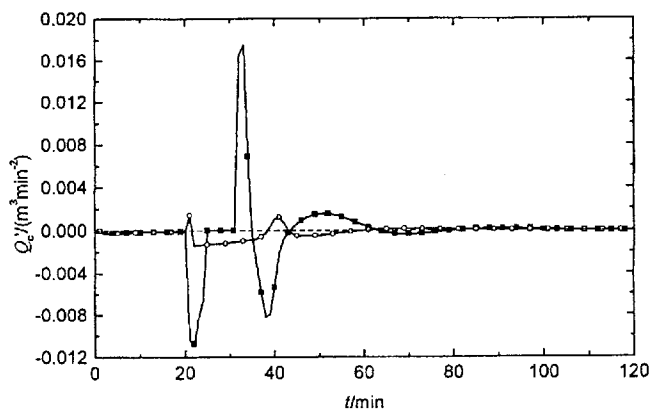


Fig. 7. 1DOF configuration - control input derivative responses to references for $\phi = 0.25$ (■) and $\phi = 25$ (○).

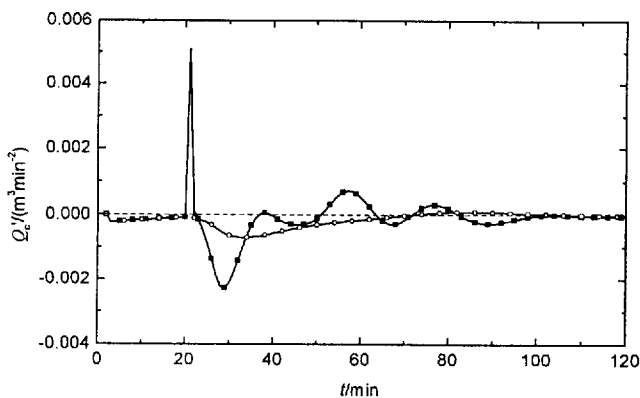


Fig. 8. 2DOF configuration – control input derivative responses to references for $\varphi = 0.25$ (■) and $\varphi = 25$ (○).

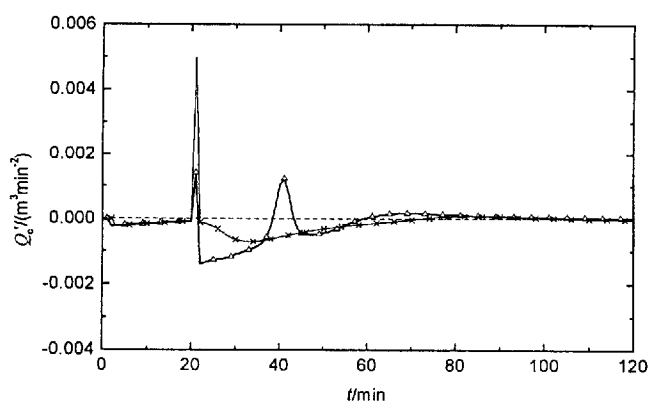


Fig. 9. Comparison of control input derivative responses in 1DOF (Δ) and 2DOF (\times) configurations for $\varphi = 25$.

At the beginning of the identification procedure (adaptation phase), a P-controller was used. Practical experiences prove that an adaptation phase for n estimated parameters needs approximately $5 \times n - 8 \times n$ identification steps. Here, the P-controller was used in 20 steps for three estimated parameters.

The controlled output, and control input derivative responses to references are shown in Figs. 4–9. The control input derivative responses in Figs. 7 and 8 demonstrate the influence of the coefficient φ (for $\mu = 1$) upon the responses. An increasing value of φ strongly reduces the control input derivatives. This fact is important with regard to a possible sensitivity of the controlled process to control input changes.

The comparison between controlled output responses in Fig. 6 shows that the control in the 2DOF configuration provides the responses of a better quality without any overshoots to step references.

In addition, the presence of the integrating part in the controller transfer function enables step load disturbance attenuation. The controlled output response in the 2DOF configuration to the step reference $w(t) = 4$ K and step disturbance $v(t) = \Delta T_{rr}(t) = \pm 2$ K is shown in Fig. 10. The control result in the presence of the random disturbance $v(t)$ is in Fig. 11.

CONCLUSION

A possible approach to the control design of nonlinear processes was proposed in this paper. The presented strategy enables to create an effective control algorithm. This algorithm is based on an alternative external linear model, which corresponds to the original nonlinear model of the process. To derive the admissible controllers in the 1DOF and 2DOF control system configurations, the polynomial approach and the LQ

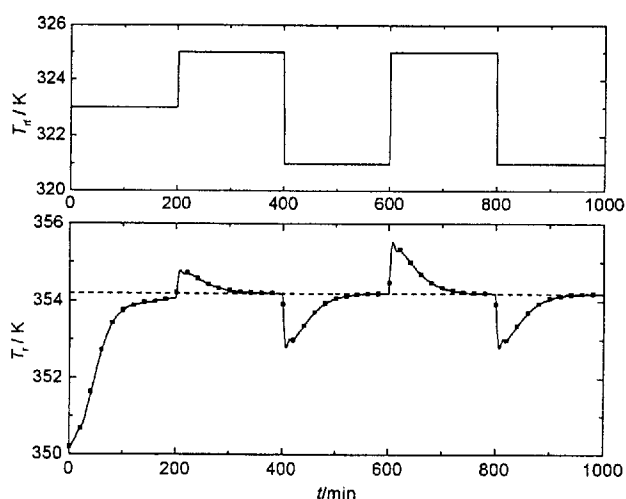


Fig. 10. Controlled temperature response (■) to the step reference value (---) and step load disturbance (—) in the 2DOF configuration.

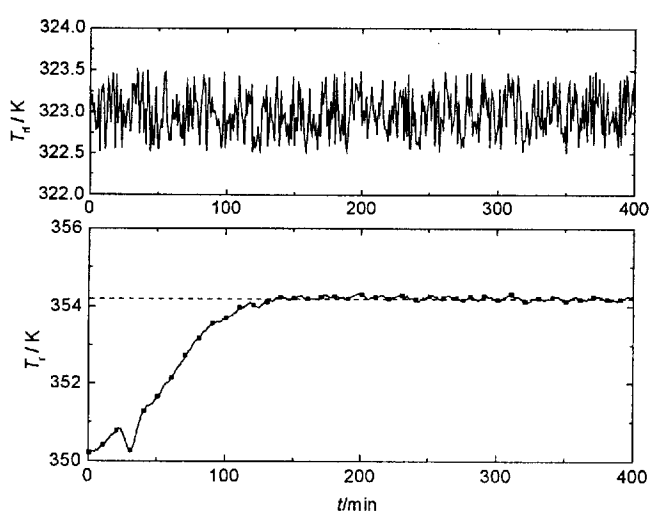


Fig. 11. Controlled temperature response (■) to the step reference value (---) in the presence of random disturbance (—).

control technique are used. The parameters of resulting continuous-time controllers are periodically readjusted according to recursively estimated parameters of an external linear model. The controller parameters can be tuned by the single selectable parameter φ .

Both resulting control systems have been tested and compared by computer simulation on the nonlinear model of a CSTR. The results demonstrate the applicability of the presented control strategy. It can be deduced that the described adaptive strategy is also suitable for other technological processes.

Acknowledgements. This work was supported in part by the Ministry of Education of the Czech Republic under the Grant MSM 2811 00001, by the Grant Agency of the Czech Republic under Grants No. 102/03/0070 and No. 102/02/0204, and by the Scientific Grant Agency of the Slovak Republic under Grants No. 1/1046/04 and 1/0135/03.

SYMBOLS

A_h	heat exchange surface area	m^2
c	molar concentration	$mol\ m^{-3}$
c_p	specific heat capacity	$J\ kg^{-1}\ K^{-1}$
E	activation energy	$J\ mol^{-1}$
h	reaction enthalpy	$J\ mol^{-1}$
Q	volumetric flow rate	$m^3\ min^{-1}$
t	time	min
t_k	discrete time intervals	min
T	temperature	K
u	control variable	
U	heat transfer coefficient	$W\ m^{-2}\ K^{-1}$
v	load disturbance	
V	volume	m^3
w	reference signal	
y	controlled variable	

Greek Letters

φ	weighting coefficient	
μ	weighting coefficient	
ρ	density	$kg\ m^{-3}$
τ_s	sampling period	min

Superscripts

s	steady-state
*	conjugate polynomial

Subscripts

c	cooling medium
r	reaction mixture
f	filtered variable, feed value

REFERENCES

1. Wahlberg, B., *Automatica* 26, 167 (1990).
2. Grimble, M. J., *Robust Industrial Control. Optimal Design Approach for Polynomial Systems*. Prentice-Hall, London, 2000.
3. Kučera, V. and Šebek, M., *Kybernetika* 20, 177 (1984).
4. Kučera, V., *Automatica* 29, 1361 (1993).
5. Dostál, P. and Bobál, V., in *Proceedings of the 7th IEEE Mediterranean Conference on Control and Automation*, p. 667. Haifa, Israel, 1999.
6. Dostál, P., Bobál, V., and Blaha, M., in *Proceedings of the IFAC Workshop on Adaptation and Learning in Control and Signal Processing ALCOSP 2001*, p. 407. Cernobbio-Como, Italy, 2001.
7. Dostál, P., Bobál, V., and Gazdoš, F., in *Proceedings of the 10th IEEE Mediterranean Conference on Control and Automation MED 2002*. Lisboa, Portugal, CD ROM-No. TUA2-6, 2002.
8. Ingham, J., Dunn, I. J., Heinzle, E., and Přenosil, J. E., *Chemical Engineering Dynamics. Modelling with PC Simulation*. VCH Verlagsgesellschaft, Weinheim, 1994.
9. Luyben, W. L., *Process Modelling, Simulation and Control for Chemical Engineers*. McGraw-Hill, New York, 1989.
10. Kulhavý, R. and Kárný, M., in *Proceedings of the 9th IFAC World Congress*, Vol. 10, p. 78. Budapest, Hungary, 1984.