Dimensionless Characteristics of Centrifugal Pump*

P. TIMÁR

Department of Chemical and Biochemical Engineering, Faculty of Chemical and Food Technology, Slovak University of Technology, SK-812 37 Bratislava
e-mail: pavel.timar@stuba.sk

Received 1 April 2005

Centrifugal pump characteristics (*i.e.* specific energy gH, power P, and efficiency η curves plotted *vs.* flow capacity Q) at different pump operating speeds are important to the successful operation of pumps in various areas of chemical technology. In the present paper the pump characteristics are treated in a dimensionless form. The main advantage of this approach is the fact that only one characteristic curve is used for all possible operating speeds of the tested pump in the frame of suitable dimensionless numbers. Also, the obtained relation between dimensionless numbers allows to estimate the main pump characteristics in the case when the properties (ρ and μ) of the liquid were varied.

Dimensional analysis is a useful and powerful tool applied mainly for investigation of complex problems. In recent years, *Zlokarnik* [1], *Szirtes* [2], and *Barenblatt* [3] used this approach for solving various technical or physical problems.

According to the authors, first it is necessary to determine all variables connected with the problem solved. This process is finished with establishing the dependent variables. In order to complete the dimensional analysis successfully, the most important condition is to find all variables influencing the system studied (neither exceeding nor missing variables). Based on the list of influencing variables, it is possible to construct the so-called dimensional matrix [1]. This matrix is divided into two parts: quadratic core matrix and residual matrix. By rearranging the core matrix to the unity matrix and using the same arithmetic operations for the residual matrix, one gets dimensionless numbers corresponding to the system studied.

During determination of relevant variables one has to keep in mind that only one dependent variable can be chosen for each individual problem. In the case of evaluation of centrifugal pump characteristics, the chosen variables are the specific energy of pump (gH) and the pump efficiency (η). Then, all independent variables influencing the target variable should be determined and the so-called relevance list of variables must be formed. For the chosen centrifugal pump the following independent variables are given in the relevance list: n – rotational speed of the pump, Q – volumetric flow rate, ρ – density, and μ – dynamic viscosity of the pumped liquid. Besides this, different geometric quantities of the pump should be taken into account: D – diameter of the pump impeller, L_1, L_2, ..., L_n – various length scales describing the pump geometry.

It is clear that only one length parameter will be important in a future analysis, because all other lengths will create simplexes of geometrical similarity. Hence, diameter of the pump impeller D was chosen as a characteristic length in this case. Mathematical formulation of the functional dependence between target variable and independent variables for the centrifugal pump is given by the following equations

$$F\,(gH, Q, \mu, \rho, D, n) = 0 \qquad (1)$$

$$G\,(\eta, Q, \mu, \rho, D, n) = 0 \qquad (2)$$

Based on the aforementioned, the dimensional matrix [1] consisting of a core and a residual matrix can be expressed as

$$\begin{array}{c|ccc|ccc} & \rho & D & n & gH & Q & \mu \\ m/\text{kg} & 1 & 0 & 0 & 0 & 0 & 1 \\ l/\text{m} & -3 & 1 & 0 & 2 & 3 & -1 \\ t/\text{s} & 0 & 0 & -1 & -2 & -1 & -1 \\ & & \text{core matrix} & & & \text{residual matrix} & \end{array} \qquad (3)$$

where m is the mass, l the length, and t the time.

It should be noted here that each variable involved in the residual matrix would occur in the resulting dimensionless numbers separately. If, for example, the rotational speed of a pump impeller n will be involved only in one dimensionless variable, it is needful

*Presented at the 32nd International Conference of the Slovak Society of Chemical Engineering, Tatranské Matliare, 23—27 May 2005.

to interchange column n by column μ in the dimensional matrix (*3*). The dimensional matrix reveals that for 6 dimensional variables only 3 dimensions (mass, length, and time) exist. Then, maximally 3 independent dimensionless variables may be obtained. To obtain these dimensionless numbers it is necessary to recalculate the above-mentioned matrix (*3*) by linear operations, the result of which is to transform the core matrix to the unity matrix. By doing so one gets

$$\begin{array}{r|ccc|ccc} & \rho & D & n & gH & Q & \mu \\ m/\mathrm{kg} & 1 & 0 & 0 & 0 & 0 & 1 \\ l/\mathrm{m} & 0 & 1 & 0 & 2 & 3 & 2 \\ t/\mathrm{s} & 0 & 0 & 1 & 2 & 1 & 1 \\ & \multicolumn{3}{c|}{\text{core matrix}} & \multicolumn{3}{c}{\text{residual matrix}} \end{array} \qquad (4)$$

Thus, according to the method of author [1] following dimensionless numbers resulting from the dimensional analysis may (matrix (*4*)) now be defined

$$\pi_1 = \frac{gH}{n^2D^2} \qquad (5)$$

$$\pi_2 = \frac{Q}{nD^3} \qquad (6)$$

$$\pi_3 = \frac{\mu}{\rho n D^2} \qquad (7)$$

The dimensionless numbers defined by eqs (*5*) and (*6*) are the so-called head coefficient and the flow coefficient of the centrifugal pump, respectively. The reciprocal value of the remaining dimensionless number π_3 is in the literature [1, 2] the well-known Reynolds number.

Following functional relations between dimensionless variables may subsequently be written

$$\pi_1 = F_1\left(\pi_2, \pi_3, \frac{L_1}{D}, \frac{L_2}{D}, \frac{L_3}{D}, \dots, \frac{L_n}{D}\right) \qquad (8)$$

$$\eta = F_2\left(\pi_2, \pi_3, \frac{L_1}{D}, \frac{L_2}{D}, \frac{L_3}{D}, \dots, \frac{L_n}{D}\right) \qquad (9)$$

If only one centrifugal pump or a series of geometrically similar pumps are taken into account (geometrical simplexes are the same), the geometrical simplexes $\frac{L_1}{D}, \frac{L_2}{D}, \frac{L_3}{D}, \dots, \frac{L_n}{D}$ remain constant and can be excluded from the relevance list, because their equality is fulfilled automatically. Then, simple relationships between dimensionless numbers are obtained

$$\pi_1 = \frac{gH}{n^2D^2} = F_1\left(\frac{Q}{nD^3}, Re\right) \qquad (10)$$

$$\eta = F_2\left(\frac{Q}{nD^3}, Re\right) \qquad (11)$$

The final form of functions F_1 and F_2 can be obtained only on the basis of experimental measurements.

EXPERIMENTAL

Experiments have been carried out with a laboratory one-stage centrifugal pump having the impeller diameter $D = 220$ mm. This equipment was used to deliver water under laboratory conditions (temperature 18 ℃). The centrifugal pump was equipped with a drive, which was able to change continuously the rotational speed of the pump. The volumetric flow rate was controlled by a valve and measured by a rotameter. A differential mercury manometer was used to measure the pressure difference between output and input sides of the centrifugal pump. The rotational speed corresponded to the speed of electric motor and during experiments was changed from 1200 min^{-1} to 2000 min^{-1}. All measured data were transformed into the form of dimensionless numbers according to eqns (*5*—*7*).

RESULTS AND DISCUSSION

Fig. 1 is the basic characteristic of the centrifugal pump: the curve head on dimensionless number π_1 connected with specific energy of centrifugal pump *vs.* dimensionless number π_2 including liquid flow rate. We can see that in the space of dimensionless variables defined by eqns (*5*) and (*6*) only one curve exists for all tested rotational speeds n. Variation of the pump impeller rotational speed caused changes of the Reynolds number (*7*) ranging from 410 000 to 680 000. These changes have no influence on the pump characteristic depicted in Fig. 1.

A similar picture can be seen for the variation of the pump efficiency with the volumetric flow rate of the liquid. This dependence is shown in Fig. 2 in the frame of derived dimensionless numbers. Only one curve for all measurements of the pump efficiency at any rotational speed (Reynolds number) exists. This

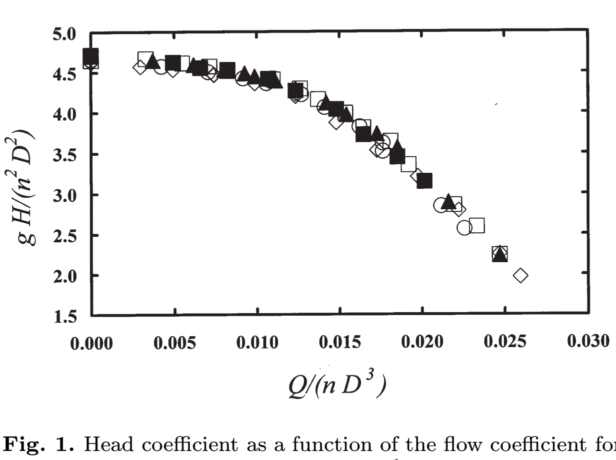

Fig. 1. Head coefficient as a function of the flow coefficient for centrifugal pump. ◇ 2000 min^{-1}, $Re = 681\ 800$; □ 1800 min^{-1}, $Re = 613\ 600$; ▲ 1600 min^{-1}, $Re = 545\ 500$; ○ 1400 min^{-1}, $Re = 477\ 300$; ■ 1200 min^{-1}, $Re = 409\ 100$.

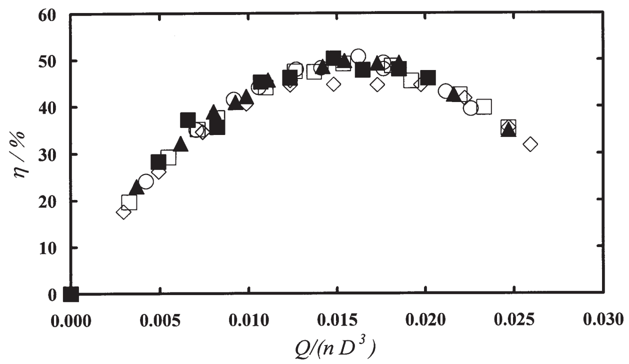

Fig. 2. Efficiency as a function of the flow coefficient for centrifugal pump. ◇ 2000 min^{-1}, *Re* = 681 800; □ 1800 min^{-1}, *Re* = 613 600; ▲ 1600 min^{-1}, *Re* = 545 500; ○ 1400 min^{-1}, *Re* = 477 300; ■ 1200 min^{-1}, *Re* = 409 100.

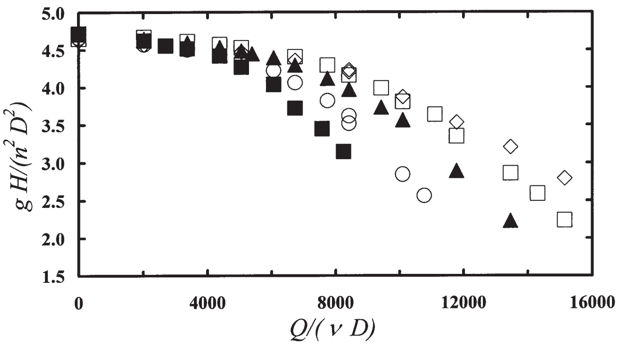

Fig. 4. Head coefficient as a function of the liquid viscosity. ◇ 2000 min^{-1}, *Re* = 681 800; □ 1800 min^{-1}, *Re* = 613 600; ▲ 1600 min^{-1}, *Re* = 545 500; ○ 1400 min^{-1}, *Re* = 477 300; ■ 1200 min^{-1}, *Re* = 409 100.

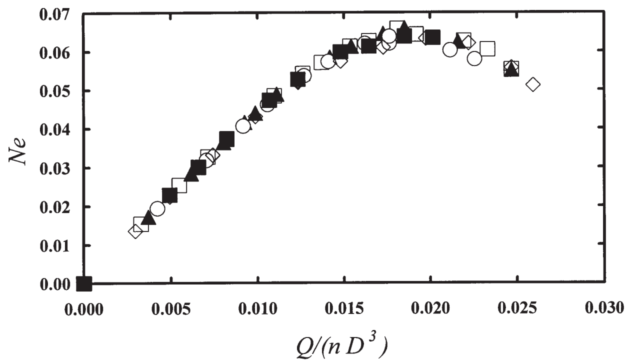

Fig. 3. Output power as a function of the flow coefficient for centrifugal pump. ◇ 2000 min^{-1}, *Re* = 681 800; □ 1800 min^{-1}, *Re* = 613 600; ▲ 1600 min^{-1}, *Re* = 545 500; ○ 1400 min^{-1}, *Re* = 477 300; ■ 1200 min^{-1}, *Re* = 409 100.

conclusion, however, can be drawn only for the range of measured Reynolds numbers mentioned above. The maximum efficiency of the tested pump was approximately 50 %. At this value the flow coefficient was about 0.015.

By multiplying the dimensionless criteria π_1 and π_2, one obtains the so-called power coefficient of the pump, a further useful characteristic of the centrifugal pump. This new dimensionless variable is known as the Newton number, *Ne* [1, 2].

$$\pi_1 \cdot \pi_2 = Ne = \frac{gH}{n^2D^2}\frac{Q}{nD^3}\frac{\rho}{\rho} = \\ = \frac{P}{\rho n^3 D^5} = F_3\left(\frac{Q}{nD^3}, Re\right) \qquad (12)$$

The plot of the Newton number *vs.* the flow coefficient is drawn in Fig. 3. From this figure it is evident that the Newton number is a function of the flow coefficient and the drawn curve exhibits a maximum similar to that of the efficiency curve depicted in Fig. 2. The maximum power output of the tested pump is located between the flow coefficient values from 0.015 to 0.020 for the whole range of rotational speeds of pump impeller. Up to this maximum point the dependence between dimensionless numbers is linear.

To estimate the pump characteristics for the transport of another liquid it is useful to rearrange the derived dimensionless numbers in such a way that the properties of the liquid are present explicitly in the dimensionless number with the flow rate of the liquid. For this purpose, a product of pump flow coefficient defined by eqn (*6*) and the Reynolds number is suitable

$$\frac{Q}{nD^3}\frac{\rho n D^2}{\mu} = \frac{Q\rho}{\mu D} = \frac{Q}{\nu D} = \pi_{\mathrm{LIQUID}} \qquad (13)$$

Variation of the head coefficient (*5*) and the new dimensionless number π_{LIQUID} defined by eqn (*13*) is depicted in Fig. 4. Estimation of pump characteristics for a new delivered liquid is possible, if the kinematic viscosity of liquid is known.

SYMBOLS

D	diameter of the pump impeller	m
F, G	function	
gH	specific energy of the centrifugal pump	$m^2\ s^{-2}$
$L_1, L_2, \ldots, L_n$	length scales describing the pump geometry	m
n	rotational speed of the pump impeller	s^{-1}
P	output power of the pump	$kg\ m^2\ s^{-3}$
Q	volumetric flow rate of the liquid	$m^3\ s^{-1}$
Re	Reynolds number (= $(\rho n D^2)/\mu$, eqn (*7*))	
Ne	Newton number (= $P/(\rho n^3 D^5)$, eqn (*12*))	

Greek Letters

ρ	density of the liquid	$kg\ m^{-3}$
μ	dynamic viscosity of the liquid	$kg\ m^{-1}\ s^{-1}$

ν	kinematic viscosity	$m^2\ s^{-1}$
η	pump efficiency	%
π	dimensionless number	

REFERENCES

1. Zlokarnik, M., *Scale-Up in Chemical Engineering.* Wiley—VCH, Weinheim, 2002.
2. Szirtes, T., *Applied Dimensional Analysis and Modeling.* McGraw-Hill, New York, 1997.
3. Barenblatt, G. I., *Dimensional Analysis and Intermediate Asymptotics.* Cambridge University Press, Cambridge, 1996.